

# Optimizing Elliptical Tank Shape Based on Real-Coded Genetic Algorithm

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**Abstract:** An elliptical tank cross-section is formulated to explore an optimization method, based on a real-coded genetic algorithm to enhance the roll stability limit of a tank vehicle. The genetic algorithm shape optimization problem is applied to minimize the overturning moment imposed on the vehicle due to CG height of the liquid load, and lateral acceleration and cargo load shift. The minimization process is performed under some major constraints such as cross-sectional area, overall height and width. The magnitudes of lateral and vertical translation of the cargo within the proposed optimal cross section under a constant lateral acceleration field are compared with those obtained with currently used elliptical cross-sections to demonstrate the performance potentials of the optimal shapes. The comparison of vehicle overturning moment revealed that the proposed optimal tank geometry is approximately 12% higher than the tank vehicle equipped with currently used elliptical and circular cross section tanks.

**Keywords:** Genetic Algorithm, Partially Filled Tanks, Roll Stability, Sloshing

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## 1 INTRODUCTION

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Rollover accidents occur in commercial vehicles which carrying the liquid cargo, where it may cause several hazards for safety of all motorists and the environment. These hazards may be related to the size and the contents of the liquid cargo such as gasoline, fuel, and ammonia. The sloshing frequencies and the magnitude of the sloshing forces, caused by moving liquid within the partially filled tank, are considered in terms of tank parameters. The directional stability limits of partially-filled liquid cargo vehicles are known to be lower than those of conventional freight vehicles.

The forces and moments, arising from a steering and braking manoeuvre, could lead to considerable dynamic load shift. Liquid load shift occurs in the roll and pitch planes due to sloshing of the liquid cargo within the partially-filled tank. The additional overturning moment caused by the dynamical load shift affects the directional stability of the vehicle in an adverse manner and may pose unreasonable risk to highway safety and the environment when dangerous fluid are hauled [1]. General purpose tanks are often designed with high capacity to transport fluids effectively. A partial-filled condition may also occur in vehicles employed in the local delivery of fuel oils. Previously reported studies have established that the magnitude of the liquid slosh under steady turning and transient steering manoeuvres and thus the roll stability limit, is strongly affected by the tank geometry, exclusively the cross-sections [2].

The design and stability analysis of vehicles which transport liquid, involves determination of liquid load developed during a specified vehicle manoeuvre. The most important factors influence the magnitude of load, i.e. liquid forces and overturning moment, are the geometrical shapes of the container, level, character of input acceleration, and fill level of the liquid. A large number of reported studies, associated with performance of elliptical tank vehicles, aim to assess directly the stability of the system by identifying the overturning limit or rollover threshold acceleration for different tank shapes and fill levels [3].

The circular tank cross-sections employed in general purpose liquid products transportation, yield high center of mass location, but considerably less lateral load transfer with low fill volumes under a steady turning lateral acceleration field. The modified oval tanks yield lower centre of gravity (CG) height and relatively larger lateral load transfer with low fill volumes in comparison with circular tanks. Under high fill volumes, the modified oval tanks exhibit higher roll stability in comparison with circular tanks [4-6]. This is mostly attributed to lower CG height associated with the modified oval geometry. A modified oval geometry may be desirable for high fill volumes.

The roll stability limits of partly-filled tank trucks over a wide range of fill volumes may be enhanced through the selection of optimal tank geometry, such that an adequate compromise between the CG height and lateral fluid slosh may be attained. Mallikarjunarao [7] proposed a semi-rectangular tank profile to achieve lower CG height by varying the top and side radius as well as the blend radius of the conventional modified oval tank (MC306 A1) profile. Richardson [8] proposed a low profile modified rectangular tank to achieve lower CG height. Klingenberg et al. [9] presented a new concept of tank truck configuration to enhance the roll stability of the vehicle by reducing the tank height and increasing the cross-sectional width.

The effects of liquid load shift in the roll plane were ignored in these investigations. Popov et al. [10], [11] investigated the optimal shapes of rectangular and elliptical road containers by minimizing the overturning moment caused by the liquid load. These methods are so complicated and indicate different manner in high and low filling conditions which demonstrates the importance of presenting a method for determining elliptical tank shape.

In this study, the elliptical tank geometry is formulated to describe various commonly used tank shapes. Also a parametric optimization is performed to minimize the CG height and magnitude of the overturning moment caused by lateral cargo movement under a wide range of fill volumes. The resulting optimal tank geometry is used to study the parametric effects due to container height, lateral acceleration, and fill level in containers with unrestricted and fixed cross sectional area. To investigate the optimal shape of the container in the case of a fixed cross-sectional area container, an optimization method is developed. The genetic algorithm is applied to derive optimal tank geometry subject in various limit constraints. The resulting optimal tank geometry is assessed in view of its CG height and lateral load shift, where the results are compared with those of conventional circular cross-sections.

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## 2 QUASI STATIC OVERTURNING MOMENT FORMULATION

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Overturning moments, resulting from lateral and vertical shifting of liquid cargo, represent potential contributors to vehicle rollover [12-16]. Steady state shifting of the liquid cargo CG has been modeled by different approaches, including geometrical modeling and experimental methods [17]. However, no straightforward formulation has been proposed for the calculation of the overturning moments due to the quasi static shifting of the cargo. When a partially filled tank truck is subjected to a constant lateral acceleration, the

free surface of the liquid inclines at a slope  $\alpha$ . This slope is numerically equals to the lateral acceleration, expressed in terms of the gravitational constant as  $g_x$ . It can be shown, through professional drawing software, the liquid free surface inclines so as to remain tangential to an ellipse that has the same aspect ratio (minor axis to major axis quotient) as the tank itself, under constant lateral acceleration. Moreover, the centre of gravity (CG) of the fluid shifts along an elliptical path with such aspect ratio. All three of these ellipses (tank form, free-surface tangent, CG path) are concentric. As shown in Fig. 1, the  $x$  and  $y$  position of the liquid cargo CG depends on the dimensions of the ellipse that describes its position and angle  $\beta$ . The coordinates  $x_f$  and  $y_f$  of the fluid within the elliptical container are described by:

$$x_f = b_f \cos \beta ; y_f = a_f \sin \beta \tag{1}$$

The slope of the fluid along the corresponding ellipse is:

$$\frac{dy_f}{dx_f} = \frac{dy_f/d\beta}{dx_f/d\beta} = \frac{a_f \cos \beta}{-b_f \sin \beta} = -\left(\frac{a_f}{b_f}\right)\left(\frac{1}{\tan \beta}\right) = \alpha = g_x \tag{2}$$

where  $r$  is the axis ratio given by:

$$\left(\frac{a_f}{b_f}\right) = \left(\frac{a_t}{b_t}\right) = \left(\frac{a_c}{b_c}\right) = r \tag{3}$$

Where  $a_f, b_f, a_t, b_t, a_c$  and  $b_c$  are known from fluid CG position (Fig. 1). Substituting Eq. (3) into Eq. (2), and solving for  $\alpha$  and  $\beta$  yields:

$$\alpha = g_x = -r \left(\frac{1}{\tan \beta}\right) ; \beta = \tan^{-1} \left(\frac{-r}{g_x}\right) \tag{4}$$

so, the liquid CG position is given by:

$$x = b_c \cos \beta ; y = a_c \sin \beta \tag{5}$$

Substituting Eq. (4) into Eq. (5) yields:

$$x = b_c \cos \left( \tan^{-1} \left( \frac{-r}{g_x} \right) \right) \tag{6}$$

$$y = a_c \sin \left( \tan^{-1} \left( \frac{-r}{g_x} \right) \right) \tag{7}$$

### 3 OVERTURNING MOMENTS

When a tank is subjected to the lateral acceleration  $g_x$ , due to the load shift within the tank, overturning moment,  $M_z$ , about the tank bottom O, as it is shown in Fig. 1, is given by:

$$M_z = F_x (a_t - y) + F_y x \tag{8}$$

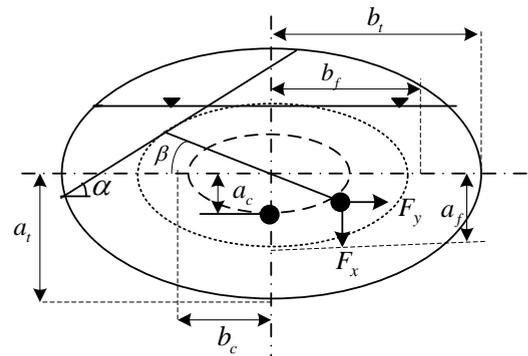


Fig. 1 Quasi-static free surface under steady lateral acceleration field

Where  $F_x = ma_x$  and  $F_y = mg_y$  are the horizontal and vertical forces, respectively, derived from subjecting the liquid cargo mass  $m$  to lateral acceleration  $a_x = 9.81g_x$  and vertical gravity acceleration  $g_y$ . Substituting Eqs. (6) and (7) into (8) yields:

$$M_z = ma_x \left( a_t - a_c \sin \left( \tan^{-1} \left( \frac{-r}{g_x} \right) \right) \right) + mg \left( b_c \cos \left( \tan^{-1} \left( \frac{-r}{g_x} \right) \right) \right) \tag{9}$$

This equation indicates that the roll moment due to the cargo shifting is a function of the liquid mass and container geometric characteristics. Eq. (9) will be used to model the dynamic response of a tank-truck to cargo sloshing.

### 4 CALCULATION OF MASS AND CG LOCATION OF A FLUID MOVING IN AN ELLIPTICAL TANK

To obtain the mass and the CG location representing the fluid motion of elliptical tank, an iterative approach is performed. Fig. 2 shows the coordinate position and the initial fluid position used in this analysis. The equation of the ellipse related to the coordinate system is:

$$\frac{x^2}{a^2} + \frac{(y-b)^2}{b^2} = 1 \tag{10}$$

which can be rewritten as:

$$y = b \left( 1 \pm \sqrt{1 - \left(\frac{x}{a}\right)^2} \right) \tag{11}$$

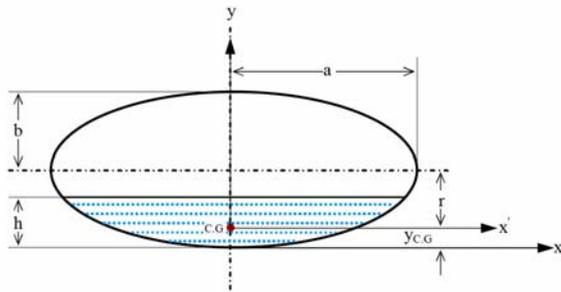


Fig. 2 Initial position of partially filled elliptical tank

Equation (10) may be rearranged as:

$$x = \pm a \sqrt{\frac{2h}{b} - \left(\frac{h}{b}\right)^2} \tag{12}$$

The value of  $x$  at a general fluid height ( $h$ ) is given by:

$$x_h = \pm a \sqrt{\frac{2h}{b} - \left(\frac{h}{b}\right)^2} \tag{13}$$

Where  $(x_1, y_1)$  and  $(x_2, y_2)$  are the points of intersection between the fluid surface line and the elliptical tank which may be found by trial and error. For finding the intersection points, the best way is sweeping a tangent to the ellipse which has an inclination angle of  $\theta$  from the upper tangent point to the lower tangent point, getting the points of intersection at each step, calculating the area and comparing it with the initial calculated area. When the right area is obtained with a certain acceptable error, the points of intersection are recorded and the values of  $x_{CG}$ ,  $y_{CG}$ ,  $R$  and  $\theta_{CG}$  may be calculated. The equation of the tangent is given by:

$$Y = mx + k \tag{14}$$

Equation (15) is achieved by solving Eqs. (10) and (23) and is represented as:

$$x^2 + \left[ \frac{2a^2 m(k-b)}{b^2 + a^2 m^2} \right] x + \left[ \frac{a^2 (k-b)^2 - b^2 a}{b^2 + a^2 m^2} \right] = 0 \tag{15}$$

As the fluid surface moves in the X-Y plane, the front area of the fluid may be used to represent the whole movement of the fluid in order to simplify the analysis. This area may be calculated from the following integration:

$$A_h = 2 \int_0^{x_h} \int_{b \left( 1 \pm \sqrt{1 - \left(\frac{x}{a}\right)^2} \right)}^h dy dx \tag{16}$$

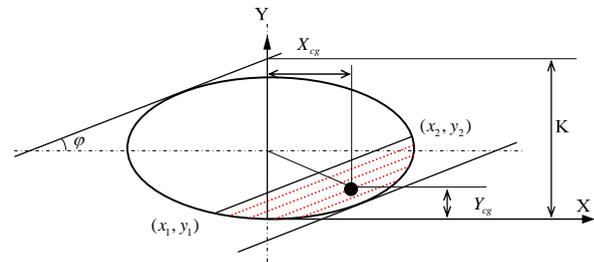


Fig. 3 Elliptical tank cross-section under lateral acceleration

If  $h < b$

$$A_h = 2 \left\{ x_h h - x_h b + \left( \frac{b}{2a} \right) \left[ x_h \sqrt{a^2 - x_h^2} + a^2 \sin^{-1} \left( \frac{x_h}{a} \right) \right] \right\} \tag{17}$$

The position of  $y$  for the fluid CG may be calculated from:

$$y_{CG} = \frac{\int_0^{x_h} \int_{b \left( 1 \pm \sqrt{1 - \left(\frac{x}{a}\right)^2} \right)}^h y dy dx}{A_h} \tag{18}$$

$$y_{CG} = \frac{1}{A_h} \left\{ h^2 x_h - 2b^2 x_h + \frac{b^2 \left[ x_h \sqrt{a^2 - x_h^2} + a^2 \sin^{-1} \left( \frac{x_h}{a} \right) \right]}{a} + \frac{b^2 x_h^3}{3a^2} \right\} \tag{19}$$

Where  $R = b - y_{CG}$ . If  $h=b$ , the fluid fills half of the ellipse with the area of  $A_h = \frac{\pi ab}{2}$ , where its CG is at:

$$R = \frac{4b}{3\pi} \tag{20}$$

If  $2b > h > b$

$$A_h = \pi ab - \alpha \tag{21}$$

Where

$$\alpha = 2 \left\{ x_h b - x_h h + \left( \frac{b}{2a} \right) \left[ x_h \sqrt{a^2 - x_h^2} + a^2 \sin^{-1} \left( \frac{x_h}{a} \right) \right] \right\}$$

$$y_{CG} = \frac{\pi ab^2 - \alpha \beta}{A_h} \tag{22}$$

The mass of liquid is calculated as:

$$M = \rho L \left( \pi ab - 2 \left\{ x_h b - x_h h + \left( \frac{b}{2a} \right) \left[ x_h \sqrt{a^2 - x_h^2} + a^2 \sin^{-1} \left( \frac{x_h}{a} \right) \right] \right\} \right) \tag{23}$$

Where  $a_x = 9.81.g_x$ ,  $a_c = y_{CG} - (h - b)$  and  $b_c = \frac{a_c}{r}$ .

Overturning moment will be calculated as follows:

$$M_z = M_h a_x \left( a - a_c \sin \left( \tan^{-1} \left( \frac{r}{g_x} \right) \right) \right) + M_h g \left( b_c \cos \left( \tan^{-1} \left( -\frac{r}{g_x} \right) \right) \right) \tag{24}$$

Fig. 4 displays variation of overturning moment for different filling percentages in tanker of Table 1, and the influence of this parameter on overturning moment. As shown in the Fig. 4, overturning moment will reduced with increase in filling percentage.

**Table 1** Main dimensions of tanker [16]

Parameter	Measurement
Trailer Unsprung Weight	1283 kg
Trailer Sprung Weight	29502 kg
Shell overall length	9.906 m
Shell overall width	2.4 m
Shell overall height	1.219 m
Weight on rear tires (empty)	47613 N/eight tires
Weight on rear tires (loaded)	165800 N/eight tires

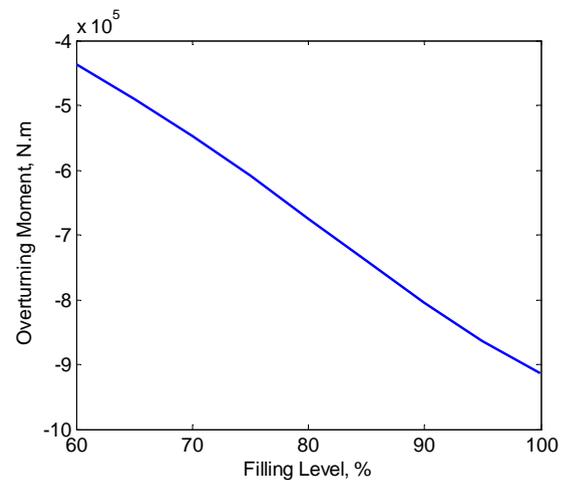


Fig. 4 Overturning moment under different parameters in elliptical tank cross-section and lateral acceleration

## 5 OPTIMIZATON PROCESS

The genetic algorithm (GA) is a non-traditional optimization method based on a Darwinian survival-of-the-fittest evolutionary theory which is a robust, random, yet global searching tool. Genetic operators such as selection, crossover and mutation are applied to individuals of a population for many generations as the method converges toward the desired solution. The basic procedure of GAs may be described as follows:

- 1- Creation of an initial population of individuals randomly.
- 2- Evaluation of all of the individuals according to the objective function. The goal of the fitness function is to numerically encode the performance of the individuals.
- 3- Select a new population based on the old population as a function of the fitness of the individual as given by the objective function.
- 4- Genetic operation (mutation and crossover): If the parents are allowed to mate, a recombination operator is employed to exchange genes between two parents to produce two children. If they are not allowed to mate, the parents are placed into the next generation unchanged. A mutation simply changes the value for a particular gene (coordinates of a control point in this case).
- 5- Evaluate these newly created individuals.
- 6- Repeat steps 3-5 (one generation) until the termination criteria have been satisfied. In most applications involving GAs, binary coding is used. However, Wright showed that real-coded GAs performs better than binary coded. A real-coded GA is used in this work.

In order to represent the optimization process, four control points (height, width, length of elliptic, and tank filling level) are limited in special constraints. To exploit the genetic algorithm properly, several parameters which indicate accuracy and convergence of the solutions such as individual numbers and mutation rate have to be determined. In this paper the value of mutation rate is 4% which is obtained after trial and error calculations.

**6 APPLYING THE REAL-CODED GENETIC ALGORITHM OPTIMATON PROCESS**

The population size represents the number of individuals in the population. Usually larger population sizes increase the amount of variation presented in the initial population where it requires more fitness evaluations. In this case, with a population size of 20 and 62 generations, an excellent solution was easily attained. Naturally, a smaller population size will introduce less generations and the calculation time will be reduced.

A comparison between all the optimized forms is shown in the Table 2. It should be noted that all cross-sections have the same area, which is the area of the conventional elliptical form used in trailer 60PRS with a cross-sectional area of 2.98 m<sup>2</sup>.

**Table 2 Optimized elliptical dimensions**

Filling conditions	50 %	60 %	70 %	80 %	90 %
optimized width size (m)	1.8	2.1	1.9	2.4	1.9
optimized height size (m)	1.25	1.3	1.35	1.1	1

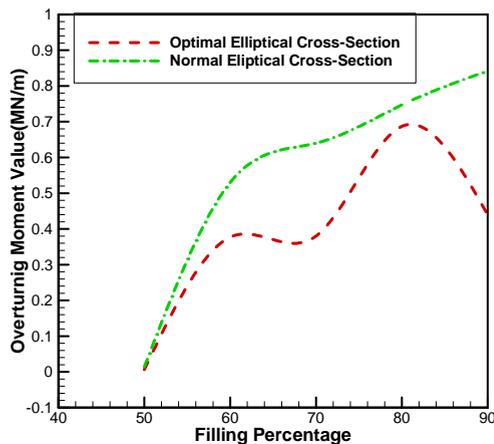


Fig. 5 Overturning moment of optimal elliptical cross section and normal elliptical cross section

The results were compared in two ways. First, the optimized forms in each percentage were compared with the elliptical form filled with the same percentage of fluid. The idea is to show that the optimized form is performing better than the elliptical tank in the case that both of them are full. The overturning moment value of all the optimized forms are compared with the conventional elliptical cross section as shown in Fig. 5. The results show the overturning moments of the optimized shapes are considerably less than the conventional elliptical cross section.

The optimization process is based on minimizing two functions which include fluid CG height ( $Y_l$ ) and overturning moment ( $M_0$ ). The goal function which is mentioned in Eq. (25), is summation of these two functions that affect simultaneously to obtain optimum cross-section. Values of weighting coefficients are equal to 0.5 according to their importance. It should be noted that those two functions behave in an opposite way. In other words, the overturning moment reduces with increasing total CG height. On the contrary, the fluid CG height decreases where the elliptical width is enhanced.

$$U(X) = Minimize [w_1M_0 + w_2Y_l], \quad w_1 = w_2 = 0.5 \quad (25)$$

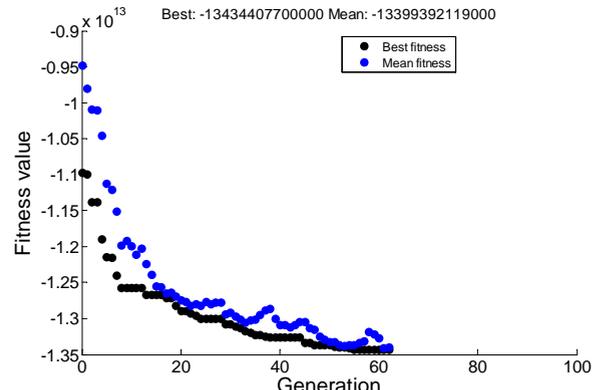


Fig. 6 Evolution of the performance for a population = 20 and generation = 62

**7 RESULTS AND DESCUSSION**

In this paper a new elliptical cross-section for different filling volume is proposed under high fill volume, as it is shown in table 2. The wider tank yield lower overturning moment due to relatively lower CG height. This study confirms the results obtained by Popov et al. [11]. This program has capability of finding excellent results for different constraints that are optional and may consider various parameters. The only constraint that was maintained constant throughout the

simulations was the area of the cross sections ( $A=2.98 \text{ m}^2$  for trailer 60PRS). The form is depicted in Fig. 5 to show the difference of overturning moment between the optimized standard elliptical shape and the conventional one. It represented that optimized value is approximately 12% higher than conventional one.

It is noted that all the new proposed tank shapes have lower centre of gravity in comparison with conventional elliptical tanks under high fill volume. Circular cross sections may be preferred for low fill volumes while a modified oval geometry may be desirable for high fill volume tanks and a new elliptical tank shape has many advantages over both conventional forms.

## 8 CONCLUSION

In this study a new method is developed to optimize the elliptical tank geometry shape to be more stable with the lowest overturning moment and fluid CG height. In this regard, a program was developed to predict the optimized shape based on conventional elliptical tank. A roll plane model of a tank vehicle has been developed and analysed for studying the lateral and vertical shifts of the fluid cargo CG. A genetic algorithm optimization program was exploited to minimise the CG height of the liquid cargo. The genetic algorithm was also used to evaluate the resulting overturning moment caused by lateral cargo movement as a function of fill volumes (50%, 60%, 70%, 80% and 90%).

The results show that the proposed method is able to reduce overturning moment of fluid tank with high accuracy. A comparison of the overturning moment of the vehicle revealed that the proposed optimal tank geometry is approximately 12% higher than the currently used elliptical and circular cross section tanks. The optimized cross-sections in different filling volumes are represented. In addition, the genetic algorithm proposed in this research is extended to obtain the better and faster convergence. As a result in the present optimization process, the mutation rate should be 4 percent.

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