Path Following of a High Speed Planning Boat Based on Gauss Pseudospectral Method

M. T. Ghorbani
Department of Mechanical Engineering,
Iran University of Science and Technology, Tehran, Iran
Email: mt_ghorbani@iust.ac.ir

Received: 27 February 2013, Revised: 11 April 2013, Accepted: 27 August 2013

Abstract: In this paper, the problem of optimal path following for a high speed planing boat is addressed. First, a nonlinear mathematical model of the boat's dynamics is derived and then the Serret-Frenet frame is presented to facilitate the path following control design. To satisfy the constraints on the states and the input controls of the boat's nonlinear dynamics and minimize both the cross tracking and heading error, a nonlinear optimal controller is formed. To solve the resulted nonlinear constrained optimal control problem, the Gauss pseudospectral method (GPM) is used to transcribe the optimal control problem into a nonlinear programming problem (NLP) by discretization of states and controls. The resulted NLP is then solved by a well-developed algorithm known as SNOPT. The results illustrate the effectiveness of the proposed approach to tackle the boat path following problem.

Keywords: Gauss Pseudospectral Method (GPM), Optimal Path Following, Optimal Control Problem, SNOPT

Reference: Ghorbani, M. T., "Path Following of a High Speed Planing Boats Based on Gauss Pseudospectral Method", Int J of Advanced Design and Manufacturing Technology, Vol. 7/ No. 1, 2014, pp. 83-89.

Biographical notes: M. T. Ghorbani received his MSc degree in Mechatronics Engineering from Sharif University of Technology in 2012. He is now a PhD candidate in Iran University of Science and Technology. His special fields of interest include Guidance, Control and Navigation of marine vehicles.

1 INTRODUCTION

The problem of path following for marine vehicles is a highly important issue. Considerable attention has been paid to the problem of path following of displacement vessels. There are some challenges in this area. The first is that the vessels are often underactuated, i.e. have fewer actuators than degrees of freedom. Advanced techniques in the field of control of under-actuated systems [1] to [4] have been suggested to path control of a 3-DoF vessel (surge, sway and yaw motion) with two independent inputs.

Another difficulty in the path following of marine surface vessels is the intrinsic physical limitations in the control inputs. The problem becomes more challenging when the controller must consider safety constraints. The two important safety constraints for marine vessels are the probabilities of slamming and deck wetness, both of which are not allowed to exceed prescribed limiting values [5]. Since the roll motion of the marine vehicle is the principal cause for the described phenomena [6], enforcing roll constraints while manoeuvring in seaways becomes an important designation in surface vessel control. To overcome the mentioned challenges, some control design methodologies have been developed.

Reference [7] used model predictive control (MPC) to control both the cross tracking and heading error by the rudder angle for an under-actuated surface vessel while considering rudder limitation and roll constraints. MPC can handle underactuated problem by combining all the objectives into a single objective function. However, due to computational complexity, the MPC applications for systems with fast dynamics (such as planning boats) are not wide spread [8]. In addition, [7] used a reduced order linear model for MPC implementation. In general, the linear models result in the loss of vital mathematical information from the dynamics of the physical system where their range of operation is small to be valid. A better choice to tackle path following problem while satisfying the input and state constraints, is nonlinear optimal control. Nonlinear optimal control satisfies any of the desirable constraints and is suitable for nonlinear system [9].

In an optimal control problem, the goal is determination of the states and controls that minimize a cost functional subject to the nonlinear dynamic constraints, the boundary condition and the inequality path constraints. There are two ways to resolve optimal control problems, namely by direct and indirect methods [10]. The indirect methods are based on Pontryagin maximum principle that transforms the optimal control problem into Euler-Lagrange equations. On the other hand, the direct methods transform the optimal control problem into a nonlinear programming problem (NLP). In this paper, to solve 4-DoF path following problem for a planning boat, a kind of direct method known as Gauss Pseudospectral Method (GPM) is addressed to transform the optimal control problem into a NLP by parameterization of the states and the controls. These parameterization techniques have an important role in convergence and accuracy of the solution and low computation time [11]. The resulted NLP is solved by a well-developed algorithm called SNOPT.

This paper is organized as follows: Section 2 presents the Gauss pseudospectral method in its most current form and provides a complete NLP, which includes both path constraints and differential dynamics in the optimal control problem formulation. In Section 3, the 4-DoF planning boat model along with the Serret-Frenet formulation is presented to facilitate the path following control design. The simulation results together with some discussions are presented in Section 4 followed by the conclusions in Section 5.

2 GAUSS PSEUDOSPECTRAL METHOD

Let's consider the following general optimal control problem and determine the state, $\mathbf{x}(t)$, and control, $\mathbf{u}(t)$, that minimize the cost functional

$$J = \Phi(\mathbf{x}(t_0), t_0, \mathbf{x}(t_f), t_f) + \int_{t_0}^{t_f} g(\mathbf{x}(t), \mathbf{u}(t), t)\, dt \tag{1}$$

Subject to the dynamic constraints

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t), \quad t \in [t_0, t_f] \tag{2}$$

The boundary conditions:

$$\mathbf{h}(\mathbf{x}(\tau_0), t_0, \mathbf{x}(\tau_f), t_f) = \mathbf{0} \tag{3}$$

The inequality path constraints:

$$\mathbf{C}(\mathbf{x}(t), \mathbf{u}(t), t) \le \mathbf{0}, \quad t \in [t_0, t_f] \tag{4}$$

Where t_0 is the fixed or free initial time and t_f is the fixed or free final time. Equations (1)-(4) are referred as the continuous Bolza problem [12]. The GPM method requires a fixed time interval, such as [−1, 1]. So the time variable is mapped to this interval via the following affine transformation.

$$\tau = \frac{2t}{t_f - t_0} - \frac{t_f + t_0}{t_f - t_0} \tag{5}$$

Rewrite the optimal control problem as:

$$J = \Phi(\mathbf{x}(\tau_0), \tau_0, \mathbf{x}(\tau_f), t_f) + \frac{t_f - t_0}{2} \int_{\tau_0}^{\tau_f} g(\mathbf{x}(\tau), \mathbf{u}(\tau); t_0, t_f) d\tau, \tag{6}$$

Subject to the constraints

$$\frac{d\mathbf{x}}{d\tau} = \frac{t_f - t_0}{2} \mathbf{f}(\mathbf{x}(\tau), \mathbf{u}(\tau), \tau; t_0, t_f) \tag{7}$$

And

$$\mathbf{h}(\mathbf{x}(\tau_0), t_0, \mathbf{x}(\tau_f), t_f) = \mathbf{0} \tag{8}$$

$$\mathbf{C}(\mathbf{x}(\tau), \mathbf{u}(\tau), \tau; t_0, t_f) \le \mathbf{0}. \tag{9}$$

Equations (5)-(9) are called the transformed continuous Bolza problem. In the GPM, this optimal control problem is discretized at some specific discretization points called the Legendre-Gauss (LG) points, and then transcribed into a nonlinear program (NLP) by approximating the states and controls using Lagrange interpolating polynomials [12]. The set of N discretization points includes $K = N - 2$ interior LG collocation points, defined as the roots of the K^{th}-degree Legendre polynomial, the initial point $\tau_0 = -1$, and the final point $\tau_f = 1$. An approximation to the state, $\mathbf{X}(\tau)$, is formed with a basis of $K+1$ Lagrange interpolating polynomials $L_i(\tau), i = 0,1,...,K,$ as follows:

$$\mathbf{x}(\tau) \approx \mathbf{X}(\tau) = \sum_{i=0}^{K} L_i(\tau) \mathbf{x}(\tau_i) \tag{10}$$

Where

$$L_i(\tau) = \prod_{j=0, j \ne i}^{K} \frac{\tau - \tau_j}{\tau_i - \tau_j} \tag{11}$$

The control is approximated using a basis of K Lagrange interpolating polynomials $L^{\dagger}_i$ as follows:

$$\mathbf{u}(\tau) \approx \mathbf{U}(\tau) = \sum_{i=1}^{K} L^{\dagger}_i(\tau) \mathbf{U}(\tau_i) \tag{12}$$

Where

$$L^{\dagger}_i(\tau) = \prod_{j=1, j \ne i}^{K} \frac{\tau - \tau_j}{\tau_i - \tau_j} \tag{13}$$

The continuous dynamics of Eq. (7) are then transcribed into the following set of K algebraic constraints via orthogonal collocation:

$$\sum_{i=0}^{K} D_{ki} \mathbf{X}(\tau_i) - \frac{t_0 - t_f}{2} \mathbf{f}(\mathbf{X}(\tau_k), \mathbf{U}(\tau_k), \tau_k; t_0, t_f) = \mathbf{0} \tag{14}$$

Where $k = 1,...,K$ and the differentiation matrix, D, consisting of the derivative of each Lagrange polynomial corresponding to the state at each collocation point. This matrix can be computed offline as follows:

$$D_{ki} = \dot{L}_i(\tau_k) = \sum_{\ell=0}^{K} \frac{\prod_{j=0, j \ne i, \ell}^{K} (\tau_k - \tau_j)}{\prod_{j=0, j \ne i}^{K} (\tau_i - \tau_j)} \tag{15}$$

Where $i=0,...,K$. Defining $\mathbf{X}_0 = \mathbf{X}(\tau_0)$ and $\mathbf{X}_f = \mathbf{X}(\tau_f)$, $\mathbf{X}_f$ is calculated via a Gauss quadrature.

$$\mathbf{X}_f = \mathbf{X}_0 + \frac{t_f - t_0}{2} \sum_{k=1}^{K} \omega_k \mathbf{f}(\mathbf{X}(\tau_k), \mathbf{U}(\tau_k), \tau_k; t_0, t_f) \tag{16}$$

Where ω_k are the Gauss weights? In addition, Eq. (6) can be approximated with a Gauss quadrature, resulting in

$$J = \Phi(\mathbf{X}_0, t_0, \mathbf{X}_f, t_f) + \frac{t_f - t_0}{2} \sum_{k=1}^{K} \omega_k g(\mathbf{X}_k, \mathbf{U}_k, \tau_k; t_0, t_f) \tag{17}$$

The boundary constraint is written as:

$$\mathbf{h}(\mathbf{X}_0, t_0, \mathbf{X}_f, t_f) = \mathbf{0} \quad (18)$$

The path constraint is computed at the LG points as:

$$\mathbf{C}(\mathbf{X}_k, \mathbf{U}_k, \tau_k; t_0, t_f) \le \mathbf{0}. \quad (k = 1, ..., K) \quad (19)$$

Equations (14), (16), (17), (18), and (19) form the NLP (20):

$$\text{Minimize}_{\mathbf{X}(\tau_k), \mathbf{U}(\tau_k)} \; J = \Phi(\mathbf{X}_0, t_0, \mathbf{X}_f, t_f) + \frac{t_f - t_0}{2} \sum_{k=1}^{K} \omega_k \, g(\mathbf{X}(\tau_k), \mathbf{U}(\tau_k), \tau_k)$$

$$\text{S.t.} \begin{cases} \mathbf{X}_f - \mathbf{X}_0 - \frac{t_f - t_0}{2} \sum_{i=1}^{K} \omega_i \mathbf{f}(\mathbf{X}(\tau_i), \mathbf{U}(\tau_i), \tau_i; t_0, t_f) = \mathbf{0} \\ \sum_{i=0}^{K} D_{ki} \mathbf{X}(\tau_i) - \frac{t_0 - t_f}{2} \mathbf{f}(\mathbf{X}(\tau_k), \mathbf{U}(\tau_k), \tau_k; t_0, t_f) = \mathbf{0} \\ \mathbf{h}(\mathbf{X}_0, t_0, \mathbf{X}_f, t_f) = \mathbf{0} \\ \mathbf{C}(\mathbf{X}_k, \mathbf{U}_k, \tau_k; t_0, t_f) \le \mathbf{0}, \quad (k = 1, ..., K). \end{cases} \quad (20)$$

Whose solution is an approximate solution to the continuous Bolza problem. In this paper to solve this NLP, SNOPT solver is used. The SNOPT is a software package for solving large-scale optimization problems. It is designed for problems with many thousands of constraints and variables but is best suited for problems with a moderate number of degrees of freedom (up to 2000) [13]. It helps us to solve non-convex optimization problems.

3 PLANING BOAT DYNAMICS MODEL IN FOUR DEGREES OF FREEDOM

In this section a summarized dynamics model of a high speed planning boat is presented that was previously explained by other authors in [14], [15]. Figure 1 shows a schematic view of the coordinate system used in the description of the marine craft dynamics. There are two reference frames shown in this figure. The first is the earth-fixed frame (x, y, z) and the other reference frame is the body-fixed frame (x_b, y_b, z_b) which is fixed to the hull. As seen from Fig. 1, it is considered that the vessel stern has a fixed pitch propeller which can also be oriented so that the propeller exerts the thrust force (T) on planning boat hull.

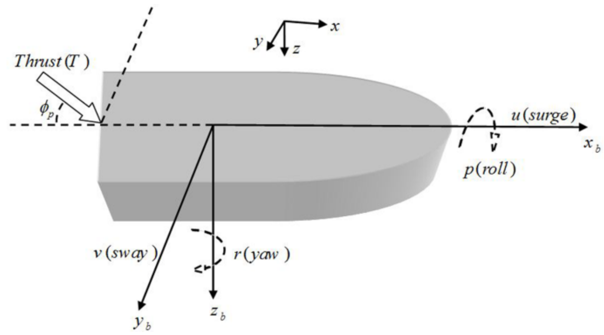

Fig. 1 Reference frames used for 4-DoF modeling of high speed craft with control forces exerted on it

In this paper, heave and pitch dynamics are neglected by assuming that the mean heave and pitch (trim) are small. The 4-DoF equations of the planning boat's dynamics may be formulated as follows [16], [17]

$$\begin{Bmatrix} m[\dot{u} - vr] \\ m[\dot{v} + ur] \\ I_{xx}^{b} \dot{p} \\ I_{zz}^{b} \dot{r} \end{Bmatrix} = \begin{Bmatrix} -\mathrm{R_T} + (T)\cos(\phi_p) \\ (T)\sin(\phi_p) \\ (T)\sin(\phi_p) \times z_p \\ -(T)\sin(\phi_p) \times x_p \end{Bmatrix} + \tau_H . \quad (21)$$

Where m is the boat's mass, $[I_{xx}^{b}, I_{zz}^{b}]$ is the vector of moment of inertia about x_b and z_b respectively. $[u, v]$ is the vector of linear surge and sway velocities and $[p, r]$ is the vector of angular roll and yaw rate in body-fixed coordinate. In the right hand of Eq. (21), $\mathrm{R_T}$ represents total (speed dependent) resistance of planning hull. To calculate this term, the boat's body plan is imported to MAXSURF software and use Savitsky pre-planning and Savitsky planning algorithm. Savitsky pre-planning algorithm is useful for estimating the resistance of a planning hull before it gets "onto the plane". On the other hand, Savitsky planning algorithm is used for estimating the resistance of planning hulls during the planning speed regime [18]. T represents thrust force and ϕ_p represents propeller turn angle relative to x_p axis. $[x_p, z_p]$ is the vector of position of propeller relative to boat's center of gravity. In addition, τ_H denotes the vector of hydrodynamic force and moments as linear functions of the velocity and acceleration components as follows:

$$\tau_H = \begin{Bmatrix} X_{\dot{u}} \dot{u} \\ Y_{\dot{v}} \dot{v} + Y_v v + Y_{\dot{p}} \dot{p} + Y_p p + Y_\phi \phi + Y_r r + Y_{\dot{r}} \dot{r} \\ K_{\dot{v}} \dot{v} + K_v v + K_{\dot{p}} \dot{p} + K_p p + K_\phi \phi + K_r r + K_{\dot{r}} \dot{r} \\ N_{\dot{v}} \dot{v} + N_v v + N_{\dot{p}} \dot{p} + N_p p + N_\phi \phi + N_r r + N_{\dot{r}} \dot{r} \end{Bmatrix} \quad (22)$$

where the coefficients are functions of speed (see [14], [16] for more detailed information). The thrust force is assumed to be fixed and compensate for the calm water resistance so that the boat nominal speed is a constant value of 10 m/s. The motion of thruster produces drag forces that slow down the boat. In addition to Eq. (21), from kinematics of the craft, there are some non-holonomic constraints as follows:

$$\begin{aligned} \dot{\phi} &= p \\ \dot{\psi} &= r\cos(\phi) \\ \dot{x} &= \mathrm{u}\times\cos(\psi) - \mathrm{v}\times(\sin\psi\times\cos\phi) \\ \dot{y} &= \mathrm{u}\times\sin(\psi) + \mathrm{v}\times(\cos\psi\times\cos\phi). \end{aligned} \tag{23}$$

Where x and y represent position of the craft in the global frame and ϕ and ψ denote roll and yaw angle of craft in the global frame respectively. Additionally, we must take into account the actuator constraints. The thrust force is limited to 7.5 kN according to Eq. (24) and the value of propeller turn angle is limited to $\mp 10^{o}$.

$$0 \le T(t) \le T_{\max} \tag{24}$$

$$\left|\phi_{\mathrm{p}}(t)\right| \le 10^{\circ} \tag{25}$$

To enforce roll motion, the roll angle and roll rate are limited as follows:

$$|\phi(t)| \le 20\,\mathrm{deg} \tag{26}$$

$$|p(t)| \le 10\frac{\mathrm{deg}}{s} \tag{27}$$

Figure 2, depicts the Serret-Frenet frame used for path following. The origin of {SF} is always the closest point on the curve C from the body-fixed {B} origin. The error dynamics based on the Serret-Frenet equations are [7]:

$$\begin{aligned} \dot{\bar{\psi}} &= \dot{\psi} - \dot{\psi}_{SF} \\ &= \frac{\kappa}{1-e\kappa}(u\sin\bar{\psi} - v\cos\bar{\psi}) + r. \end{aligned} \tag{28}$$

$$\dot{e} = u\sin\bar{\psi} + v\cos\bar{\psi} \tag{29}$$

In Eq. (29), e, is defined as the distance between the origins of {SF} and {B}, and $\bar{\psi} = \psi - \psi_{SF}$, are referred to the cross track error and heading error respectively. ψ_{SF} is the path tangential direction as shown in Fig. 2 [19], and κ is the curvature of the given path. The control objective of the path following problem is to drive e and ψ to zero. So a linear quadratic performance index is introduced as a function of e and ψ as:

$$J = \frac{1}{2}\int_{0}^{t_f}(Qe^2 + P\bar{\psi}^2) \tag{30}$$

Where Q, P are positive weight constants.

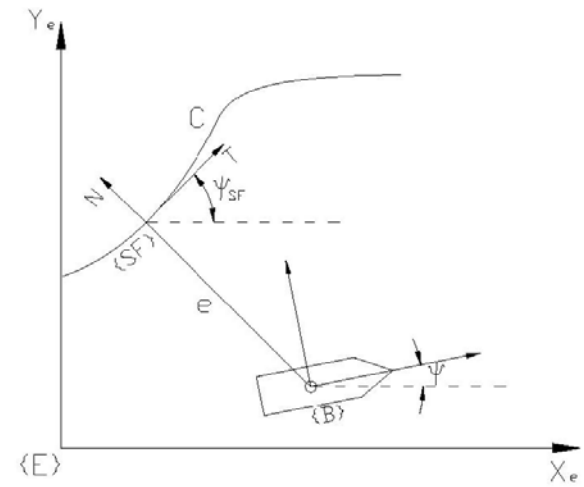

Fig. 2 Illustration of the coordination in the earth-fixed frame {E}, the ship body-fixed frame {B}, and the Serret-Frenet frame {SF} [7]

4 APPLICATION

Economy (fuel cost), safety (related to accuracy and maneuverability), and user preferences are the three major factors which play an important role in controller design. The autopilot system must penalize undesirable quantities such as:

- deviation of the controlled variables from the desired value,
- amount of control signal (rudder angle),
- amount of roll angle for safety reasons.

These demands must be translated into a performance criterion function to be minimized by an optimal control system.
The autopilot then has two main tasks:

- steady-state *track* (or *course*) *keeping*.
- *course changing* (that is, *turning* or *maneuvering*).

There are, therefore, two steering modes to be considered; *course changing* and *track keeping*. For this purpose, two scenarios are defined: one for course keeping and one for course changing. In the first scenario, a straight line reference trajectory is defined with ψ_{SF} = 45 degrees. The boat initial heading and the cross track error are assumed to be -90^o and 10 m respectively. It is also assumed that the boat's initial position is at the earth-fixed origin. Fig. 3, shows the desired path and the boat course for Q = 100, P = 1000. As seen, the boat heading and cross track error, become zero after an initial deviation from the desired path.

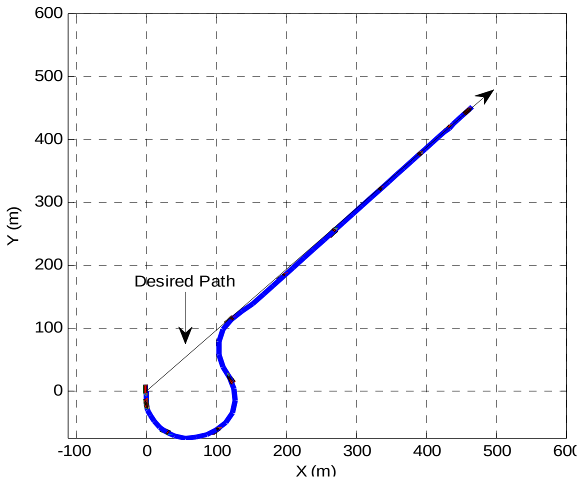

Fig. 3 The desired path and the boat course to follow a straight line

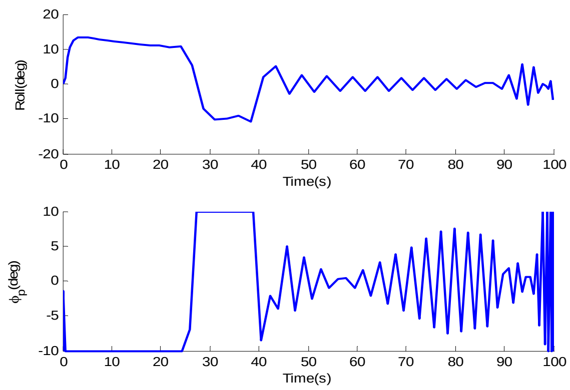

Fig. 4 The roll and the propeller turn angle during straight line following

The roll and the propeller turn angle are shown in Fig. 4. The amounts of roll and propeller turn angle do not exceed their corresponding limit. The mean computation time for the 60-node solution for the above simulation is approximately 23 seconds.

To verify the ability of the proposed path following controller for harder situations, in which both cross track and heading error are not zero and the desired heading changes with time, we define a circular path with ψ_{SF} = 0.033 πt. The boat initial heading and cross track error in this scenario are 90 deg and 10 m respectively.

Figure 5 shows the boat and desired heading to be followed in the simulation. As seen in the figure, the boat initiates the course by 90 deg heading, afterwards the heading error decreases to reach the desired path.

The roll and the propeller turn angle are shown in Fig. 6. Again, the roll and propeller turn angle values do not exceed their corresponding limit.

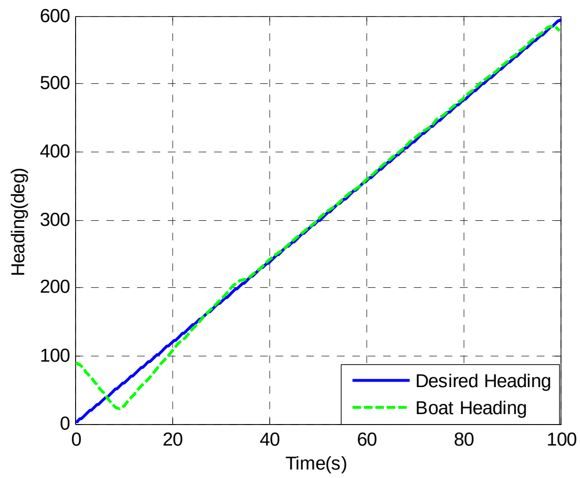

Fig. 5 The desired path and the boat course to follow a circular path

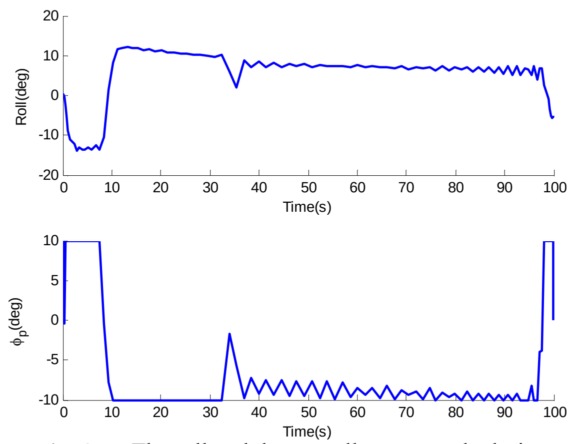

Fig. 6 The roll and the propeller turn angle during circular path following

Finally, Fig. 7, depicts the course of the boat in the last scenario. The controller ability in the second scenario shows its great performance of LQR based feedback controller, since both the reference and actual trajectory of the vessel is changing with time and both the values of roll angle and the deflection of the rudder are permissible.

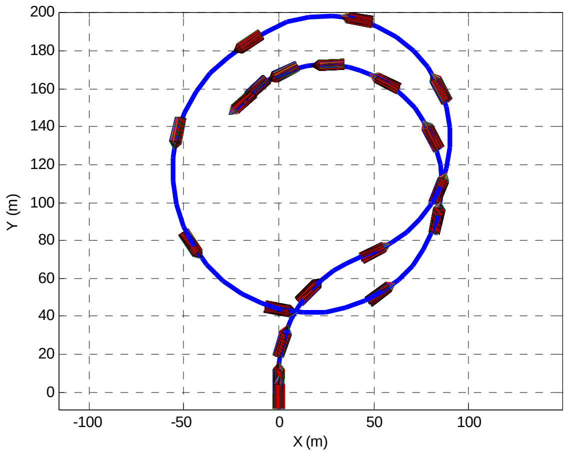

Fig. 7 Boat circular path

5 CONCLUSION

In this paper the problem of path following for 4-DoF dynamics of a planning boat is defined as an optimal control problem. The objective of the optimal control problem is to drive the cross track and heading error between the vessel and desired path in the Serret-Frenet frame to zero. Because of intricate dynamics of high-speed planning boats, hard constraints on states and controls, convergence towards a global optimum is difficult to achieve with traditional methods. The Gauss pseudospectral method is used to transcribe optimal control problem to (NLP). SNOPT solver also helped us to solve the resulted expensive NLP. Two case studies of straight and circular path has been examined and showed that the proposed method can handle the path following problem of high speed planning crafts.

REFERENCES

[1] Do, K., Pan, J., "Under actuated ships follow smooth paths with integral actions and without velocity measurements for feedback: theory and experiments", IEEE transactions on control systems technology, Vol. 14, No. 2, 2006, pp. 308-322.

[2] Fossen, T., Breivik, M., and Skjetne, R., "Line-of-sight path following of under actuated marine craft", Proceeding of the Sixth IFAC Conference Maneuvering and Control of Marine Crafts, 2003, pp. 244-249.

[3] Jiang, Z., "Global tracking control of underactuated ships by Lyapunov's direct method", Automatica, Vol. 38, 2002, pp. 301-309.

[4] Lefeber, E., Pettersen, K., and Nijmeijer, H., "Tracking control of an under actuated ship", IEEE Transactions on Control Systems Technology, Vol. 11, 2003, pp. 52-61.

[5] Avgouleas, K., "Optimal ship routing", MSc. thesis, Dept. Mech. Eng., Massachusetts Univ., Massachusetts, USA, 2008.

[6] Fossen, T., Marine Control Systems. Marine Cybernetics, 2002.

[7] Li, Z., Sun, J., and Oh, S., "Path following for marine surface vessels with rudder and roll constraints: an MPC approach", American Control Conference, 2009, pp. 3611-3616.

[8] Ghaemi, R. "Robust Model Based Control of Constrained Systems", PhD. dissertation, Dept. Elec. Eng., Univ. Michigan, 2010.

[9] Cimen, T., Banks, S. P., "Nonlinear optimal tracking control with application to super-tankers for autopilot design", Automatica, Vol. 40, 2004, pp. 1845-1863.

[10] Betts, J. T., "Survey of numerical methods for trajectory optimization", Journal of Guidance, Control, and Dynamics, Vol. 21, 1998, pp. 193-207.

[11] Benson, D., "A Gauss pseudo spectral transcription for optimal control", PhD. dissertation, Massachusetts Institute of Technology, United States, Massachusetts, 2005.

[12] Huntington, G., "Advancement and analysis of a Gauss pseudospectral transcription for optimal control problems", PhD. dissertation, Massachusetts Institute of Technology, United States, Massachusetts, 2007.

[13] Gill, P. E., et al., "SNOPT: An SQP algorithm for large-scale constrained optimization", SIAM Review, Vol. 47, 2005, pp. 99-131.

[14] Salarieh, H., Ghorbani, M. T. "Trajectory Optimization for a High Speed Planing Boat Based on Gauss Pseudo spectral Method", 2nd International Conference on Control, Instrumentation, and Automation (ICCIA 2011).

[15] Ghorbani, M. T., "Path planning and design of optimal guidance algorithm for a high speed planing craft", MSc. thesis, Dept. Mech. Eng., Sharif Univ. of Tech., Tehran, Iran, 2012.

[16] Lewandowski, E. M., The dynamics of marine craft: maneuvering and seakeeping: World Scientific, 2004.

[17] Fossen, T. I., Handbook of Marine Craft Hydrodynamics and Motion Control: John Wiley & Sons, 2011.

[18] "Hull speed User Manual", Formation Design Systems Pty Ltd, 1984-2011.

[19] Skjetne, R., Fossen, T. I., "Nonlinear maneuvering and control of ships", Proceeding of the MTS/IEEE OCEANS, 2001.