

Free Vibration Analysis of Nanoplates using Differential Transformation Method

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Abstract: In this paper, a free vibration of nano-plates is investigated considering the small scale parameter. The used rectangular nano-plate is thin and under different boundary conditions. In order to obtain the natural frequencies of the nano-plates, classical plate theory on the basis of non-local theory is used. The governing equation is solved using a semi-analytical method DTM (Differential transformation method). The results for free vibration of those plates are compared with the theoretical data published in the literature. Results show that DTM is a powerful, simple, accurate and fast method for solving equations in comparison with other methods. Non-local parameter is very effective in vibration of nano-plates and its influence is different in various boundary conditions. Influence of this parameter in simply supported-clamp boundary condition is higher than other boundary conditions.

Keywords: DTM, Free vibration, Nano-plate, Semi-analytical

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1 INTRODUCTION

Due to outstanding physical, chemical, mechanical and electronic properties [1–4], nano-sized structures [5–8] have attracted a great deal of attention in scientific community. Therefore, development of appropriate mathematical models for nanostructures is an important issue concerning application of nanostructures. A review related to the importance and modelling of vibration behaviour of various nanostructures can be found in Gibson's et al., [9]. Vibration of nanostructures has great importance in nanotechnology, where understanding vibration behaviour of nanostructures is the key step for many NEMS devices like oscillators, clocks and sensor devices.

Three approaches have been developed to model nanostructures. These approaches are (a) atomistic [10–11], (b) hybrid atomistic–continuum mechanics [12–15], and (c) continuum mechanics. Nevertheless, the first two methods involve solving a large number of equations. Hence, they have difficulties in handling systems with large length and time scales. Since performing experiments in nano-level are difficult to control and also theoretical atomistic and hybrid atomistic–continuum models are computationally expensive for relatively large scale nanostructures, the continuum [12], [13], [16] models have been proven to be important tools in the study of the nanostructures. Continuum mechanics approach is less computationally expensive than the former two approaches. It is found that continuum mechanics results are in good agreement with atomistic and hybrid approaches. Considering the continuum models for small devices, the use of traditional elasticity theory may lead to erroneous results as continuum assumption may not hold valid in the small scales.

This fact triggered development of various micro-continuum theories such as couple stress theory [17], micro-morphic theory [18], strain gradient elasticity theory [19] and non-local elasticity theory [20]. Among these theories, non-local elasticity theory has been widely applied to various problems of physics. Such theories contain information about the forces between atoms, and the internal length scale is introduced into the constitutive equations as a material parameter. Chen et al., [21] proved that non-local elasticity theory is consistent with the molecular dynamics [22–24]. This has made the non-local elasticity theory an efficient alternate to atomistic methods. In non-local elasticity theory, the scale effects are taken into account by considering internal size as a material parameter.

The most general form of the constitutive relation for non-local elasticity involves an integral over the whole body and therefore the governing equations become integro-differential equations (see e.g. [1–2]). Eringen [25] showed that it is possible to represent the integral

constitutive relations of nano-structures in an equivalent differential form. While, Most classical continuum theories are based on hyper elastic constitutive relations which assume that the stress at a point are functions of strains at that point, Eringen [25] presented a non-local elasticity theory to account the small scale effect by specifying the stress at a reference point as a functional of the strain field at every point in the body. Afterward, the non-local differential elasticity (or non-local stress gradient elasticity) has gained more popularity among the researchers due to its simplicity. Compared to classical continuum mechanics theories, non-local theory of Eringen has capability to predict behavior of the large nano-sized structures, while it avoids solving the large number of equations. Here, the inter-atomic forces and atomic length scales directly come to the constitutive equations as material parameters [25]. Thus, it appears that non-local continuum mechanics could potentially play an important role in future. Therefore, many papers have been published on this topic, especially for analyzing of nano-structures (see, for example, the non-local theory of longitudinal waves in an elastic circular bar [26], non-local theory solution of two collinear cracks in the functionally graded materials [27], buckling analysis of CNT based on non-local theory [5], non-local theories of beams [28–29]).

Initially Peddieson et al. [30] applied current non-local elasticity theory and studied flexural behavior of one dimensional nanostructures. Since then a large number of research activities using non-local elasticity theory have taken place. The non-local theory of elasticity has been extensively used for buckling and vibration analyses of carbon nano-tubes with the help of beam and shell theories [26], [31], [32]. There are already studies on the continuum models for vibration of carbon nanotubes (CNTs) or similar micro or nano beam like elements [33–37]. A relevant reference concerning non-local theories for bending, buckling and vibration analysis of beams is reported by Reddy [28]. Also, one can find some papers about the analysis of nano beams in [39–45]. Unlike to one-dimensional non-local theories, there are only a few studies on two-dimensional ones [40], [41], [43], [46–48]. In the continuum models used in [40], [41], [43], [47–48] only classical plate theory (CLPT) has been considered for modelling the nanoplates. These mathematical models do not take scale effect into account.

Kitipornchai et al. [40] used the continuum plate model for mechanical analysis of graphene sheets. Pradhan and Phadikar [49] presented the vibration analysis of a simply supported nano-plate based on the first order shear deformation plate theory (FSDT). Aghababaei and Reddy [50] solved bending and vibration of plate problems based on the non-local third order shear deformation plate theory (TSDT) considering the small

scale effect. So far two methodologies have been used extensively for solving the governing differential equations arising in structural analysis of non-local elastic nanostructures. These are Navier’s method [50-52] and Differential Quadrature Method (DQM) [53-55]. However it is well known that Differential transformation method (DTM) unlike to these methods can effectively handle more complex geometry, material property, boundary and/or loading conditions. The DTM is a semi analytical–numerical technique based on the Taylor series expansion method for solving differential equations. It is different from the traditional high order Taylor series method.

The Taylor series method computationally takes long time for large orders. However, with DTM, doing some simple mathematical operations on differential equations, a closed form series solution or an approximate solution can be obtained quickly. This method was first proposed by Zhou [56] in 1986 for solving both linear and nonlinear initial-value problems of electrical circuits. Later, Chen and Ho [57] developed this method for partial differential equations and Ayaz [58], [59] studied two and three dimensional differential transformation method of solution of the initial value problem for partial differential equations. Arikoglu and Ozkol [60] extended the differential transformation method to solve the integro-differential equations.

Catal [61] used DTM for free vibration analyses of both ends simply supported beam resting on elastic foundation. Recently, researchers used the DTM method successfully to handle various kinds of rotating beam problems (Kaya [62]; Ozdemir and Kaya [63]; Ozdemir and Kaya [64]). It is importance to incorporate non-local elasticity theories in the vibration analysis of nanoplates. In the present paper attempt is made to study the vibration of the nanoplates using non-local elasticity theory. The CPT has been incorporated in the analysis. DTM approach has been used to solve the governing equations for different boundary conditions.

2 MATHEMATICAL FORMULATION

2.1. Geometrical configuration

A flat, isotropic, and thin rectangular plate of length a and width b is depicted in Fig. 1. The plate has two opposite edges simply supported along y axis (i.e. along the edges $x = 0$ and $x = a$), while, the other two edges may be free, simply supported, or clamped. The Cartesian coordinate system (x, y, z) is considered to extract mathematical formulations when x and y axes are located in the undeformed mid-plane of the plate.

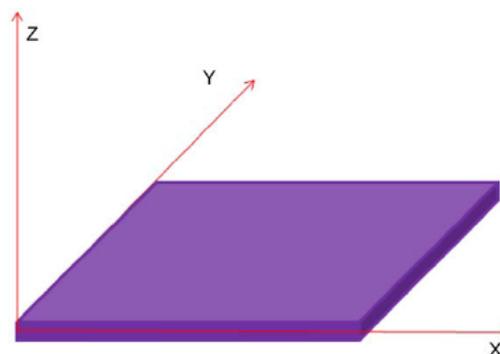


Fig. 1 Schematic view of the nanoplate [49]

2.2. Constitutive relations

According to the classical plate theory, the components of displacement of points within the plate will be characterized by:

$$u = u_0(x, y) - z \frac{\partial w_0}{\partial x} \quad v = v_0(x, y) - z \frac{\partial w_0}{\partial y} \tag{1}$$

$$w = w_0(x, y)$$

Where u , v , and w are measured in the x , y and z direction, respectively. Here, displacements with subscript 0 are the related components in the mid-plane of the plate. Using Eq. (1) and the linearized strain-displacement equations one can obtain:

$$\begin{aligned} \epsilon_{11} &= \frac{\partial u_0}{\partial x} - z \frac{\partial^2 w_0}{\partial x^2} \\ \epsilon_{22} &= \frac{\partial v_0}{\partial y} - z \frac{\partial^2 w_0}{\partial y^2} \\ \epsilon_{12} &= \frac{1}{2} \left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2z \frac{\partial^2 w_0}{\partial x \partial y} \right) \end{aligned} \tag{2}$$

In classical local elasticity theories, stress at a point depends only on the strain at that point, while in non-local elasticity theories it is assumed that the stress at a point depends on the strains at all the points of the continuum. In other words, according to this non-local theory strain at a point depends on both stress and spatial derivatives of the stress at that point. According to Eringen [14] the non-local constitutive behavior of a Hookean solid is represented by the following differential constitutive relation:

$$(1 - \mu \nabla^2) \sigma^{ij} = \sigma^l \tag{3}$$

In which, μ is the non-local parameter and σ^l the local stress tensor at a point which is related to strain by generalized Hooke’s law. Here, ∇^2 is biharmonic operator. Using above equation and Hook’s law, the plane stress constitutive relation for a non-local thin plate will be obtained as:

$$\begin{Bmatrix} \sigma_{11}^{nl} \\ \sigma_{22}^{nl} \\ \sigma_{12}^{nl} \end{Bmatrix} - \mu \nabla^2 \begin{Bmatrix} \sigma_{11}^{nl} \\ \sigma_{22}^{nl} \\ \sigma_{12}^{nl} \end{Bmatrix} = \begin{bmatrix} \frac{E}{(1-\nu^2)} & \frac{\nu E}{(1-\nu^2)} & 0 \\ \frac{\nu E}{(1-\nu^2)} & \frac{E}{(1-\nu^2)} & 0 \\ 0 & 0 & 2G \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{Bmatrix} \quad (4)$$

Where, E and ν are the Young's module and the Poisson's ratio, respectively, and G is $\frac{E}{2(1+\nu)}$.

Bending moments are obtained by integrating the in-plane stresses over the plate thickness. In the case of a homogeneous plate, the stress resultants are:

$$\begin{aligned} M_{11} &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{11} z \, dz, & M_{12} &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{12} z \, dz, \\ M_{22} &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{22} z \, dz \end{aligned} \quad (5)$$

Using Eqs. (2), (4), and (5), the form of stress resultants in non-local theory will be yield as:

$$\begin{aligned} M_{11} - \mu \nabla^2 M_{11} &= -D \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right), \\ M_{22} - \mu \nabla^2 M_{22} &= -D \left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right), \\ M_{12} - \mu \nabla^2 M_{12} &= -D(1-\nu) \left(\frac{\partial^2 w}{\partial y \partial x} \right), \end{aligned} \quad (6)$$

in which

$$D = \frac{Eh^3}{12(1-\nu^2)}$$

2.3. Equations of motion

The governing differential equations of motion for the plate without any external loading can be given in terms of the stress resultants by:

$$\begin{cases} M_{11,1} + M_{12,2} - Q_1 = 0 \\ M_{12,1} + M_{22,2} - Q_2 = 0 \\ Q_{1,1} + Q_{2,2} - P = \rho h \frac{\partial^2 w}{\partial t^2} \end{cases} \quad (7a, b, c)$$

Obtaining Q_1 and Q_2 from Eq. 7(a) and 7(b) and then substituting them in Eq. 7(c), the equation of motion for thin plates will be resulted. By using non-local stress resultants in the equation of motion, one can obtain the non-local governing equation for thin nano plates as:

$$D \left(\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) = \rho h \omega^2 (1 - \mu \nabla^2) w \quad (8)$$

Or

$$D \nabla^4 w = \rho h \omega^2 (1 - \mu \nabla^2) w \quad (9)$$

2.4. Boundary conditions

In the considered rectangular nano plate, the boundary conditions along the edges $y = 0$ and $y = b$, are simply supported and as follows:

$$M_{yy} = w = 0 \quad (10)$$

The boundary conditions along the edges $x = 0$ and $x = a$ can be free, simply supported, or clamped and as follows:

For a free edge

$$M_{xx} = M_{yx} = 0 \quad (11)$$

For a simply supported edge

$$M_{xx} = w = 0 \quad (12)$$

For a clamped edge

$$w = \frac{\partial w}{\partial x} = 0 \quad (13)$$

On the assumption of simply supported conditions at edges $y = 0$ and b , one set of solution to Eq. (9) can be given as:

$$w = w(x) \sin\left(\frac{n\pi y}{b}\right) \quad (14)$$

Applying above solution in governing equation (8), the final equation of motion for the thin rectangular Nano-plate with simply supported opposite edges will be resulted as:

$$\begin{aligned} D \left[\frac{\partial^4 w_1}{\partial x^4} - 2 \frac{n^2 \pi^2}{b^2} \frac{\partial^2 w_1}{\partial x^2} + \frac{n^4 \pi^4}{b^4} w_1 \right] = \\ \rho h \omega^2 \left[w_1 - \mu \left(\frac{\partial^2 w_1}{\partial x^2} - \frac{n^2 \pi^2}{b^2} w_1 \right) \right] \end{aligned} \quad (15)$$

To solve Eq. (15), DTM will be used. A detailed description on how to use this method is given in the following section.

2.5. DTM Solution

Basic definitions and operations of differential transformation are introduced in the following.

Differential transformation of the function $f(\eta)$ is defined as follows:

$$F(k) = \frac{1}{k!} \left[\frac{d^k f(\eta)}{d\eta^k} \right]_{\eta=\eta_0}, \tag{16}$$

In Eq. (16) $f(\eta)$ is the original function and $F(k)$ is transformed function which is called the T-function (it is also called the spectrum of the $f(\eta)$ at $\eta = \eta_0$ in the K domain). The differential inverse transformation of $F(k)$ is defined as:

$$f(\eta) = \sum_{k=0}^{\infty} F(k)(\eta - \eta_0)^k, \tag{17}$$

Combining Eqs. (16) and (17), gives:

$$f(\eta) = \sum_{k=0}^{\infty} \left[\frac{d^k f(\eta)}{d\eta^k} \right]_{\eta=\eta_0} \frac{(\eta - \eta_0)^k}{k!}, \tag{18}$$

Eq. (18) implies that the concept of the differential transformation is derived from Taylor's series expansion, but the method does not evaluate the derivatives symbolically. However, relative derivative are calculated by iterative procedure that are described by the transformed equations of the original functions. From the definitions of Eqs. (16) and (17), it is easily proven that the transformed functions comply with the basic mathematical operations shown in below. In real applications, the function $f(\eta)$ in Eq. (17) is expressed by a finite series and can be written as:

$$f(\eta) \cong \sum_{k=0}^N F(k)(\eta - \eta_0)^k, \tag{19}$$

Eq. (19) implies that $\sum_{k=N+1}^{\infty} F(k)(\eta - \eta_0)^k$ is negligibly small, where N is series size. Theorems to be used in the transformation procedure, which can be evaluated from equations (16) and (17), are given at below:

Theorem 1. If $f(\eta) = g(\eta) \pm h(\eta)$, then $F(k) = G(k) \pm H(k)$.

Theorem 2. If $f(\eta) = cg(\eta)$, then $F(k) = cG(k)$, where c is a constant.

Theorem 3. If $f(\eta) = \frac{d^n g(\eta)}{d\eta^n}$, then

$$F(k) = \frac{(k+n)!}{k!} G(k+n).$$

Theorem 4. If $f(\eta) = g(\eta)h(\eta)$, then $F(k) = \sum_{l=0}^k G(l)H(k-l)$.

Theorem 5. If $f(\eta) = \eta^n$, then $\delta_D(k-n)$ that $\delta_D(k-n) = \begin{cases} 1 & k=n \\ 0 & k \neq n \end{cases}$

Although this newly emerged method has been proved to be an efficient tool for handling nonlinear problems, the nonlinear function $h(f(\eta))$ used in other studies is restricted to the types of nonlinear polynomials and derivatives. For other type of nonlinearity such as hyperbolic function, Zhou [56] introduced the standard way to calculate its transformed function. Using transformation operation which is defined in theorem 1 to 5 and taking the differential transformation of equation 15 at $x=0$, one may obtain:

$$\begin{aligned} & (K+4)(K+3)(K+2)(K+1)F[K+4] - \\ & \left(\frac{\rho h \omega^2}{D} + \frac{n^2 \pi^2 \rho h \omega^2 \mu}{b^2 D} - \frac{n^4 \pi^4}{b^4} \right) F[K] + \\ & \left(\frac{\rho h \omega^2 \mu}{D} - 2 \frac{n^2 \pi^2}{b^2} \right) (K+2)(K+1)F[k+2] = 0 \end{aligned} \tag{20}$$

To complete the formulation, we have to discuss the boundary conditions. six combinations of boundary conditions, i.e., C-C, C-F, C-S, S-S, S-F, and F-F, where S, C and F denote the simply supported, clamped and free boundaries, respectively and may be written in the same order as:

$$f(0) = f'(0) = f(b) = f'(b) = 0 \tag{21}$$

$$f(0) = f'(0) = f''(b) = f'''(b) = 0 \tag{22}$$

$$f(0) = f'(0) = f(b) = f''(b) = 0 \tag{23}$$

$$f(0) = f''(0) = f(b) = f''(b) = 0 \tag{24}$$

$$f(0) = f''(0) = f''(b) = f'''(b) = 0 \tag{25}$$

$$f''(0) = f'''(0) = f''(b) = f'''(b) = 0 \tag{26}$$

The boundary conditions Eqs. (21) to (26) can be represented in the Differential transformation form as shown in table 1.

Using MATHEMATICA software the terms of series were obtained which some of series terms are depicted in the following.

Table 1 DTM theorems used for boundary conditions

$x = 0$		$x = b$	
Original B.C.	Transformed B.C.	Original B.C.	Transformed B.C.
$f(0) = 0$	$F(0) = 0$	$f(b) = 0$	$\sum_{k=0}^{\infty} F(k)b^k = 0$
$\frac{df}{dx}(0) = 0$	$F(1) = 0$	$\frac{df}{dx}(b) = 0$	$\sum_{k=0}^{\infty} kF(k)b^{k-1} = 0$
$\frac{d^2f}{dx^2}(0) = 0$	$F(2) = 0$	$\frac{d^2f}{dx^2}(b) = 0$	$\sum_{k=0}^{\infty} k(k-1) \times F(k)b^{k-2} = 0$
$\frac{d^3f}{dx^3}(0) = 0$	$F(3) = 0$	$\frac{d^3f}{dx^3}(b) = 0$	$\sum_{k=0}^{\infty} k(k-1)(k-2) \times F(k)b^{k-3} = 0$

$$F[4] = 0$$

$$F[5] = (-2.56551 \times 10^{32} + 436.599 \omega^2) C_1 +$$

$$(1.7546 \times 10^{16} - 2.22857 \times 10^{-15} \omega^2) C_2$$

$$F[6] = 0$$

$$F[7] = \left(\begin{matrix} -2.14354 \times 10^{48} + 3.92014 \times 10^{18} \omega^2 - \\ 4.633 \times 10^{-13} \omega^4 \end{matrix} \right) C_1 +$$

$$(1.0995 \times 10^{32} + 25.1309 \omega^2 + 2.36501 \times 10^{-30} \omega^4) C_2$$

For solving equation (20), four terms of series are needed. Two terms are derived from boundary condition, for example in S-S boundary condition, deflection and moment are zero, hence F[0] and F[2] take zero value, two other terms F[1] and F[3] which are relate to the ramp and shear conditions have unknown value, hence none zero C1 and C2 value are attributed to them. Using boundary condition in edge $x=b$, two equations are obtained. Knowing this fact that C1 and C2 must not be zero, natural frequency can be calculated.

3 COMPARISON

In order to verify the results obtained by DTM method for non-local theory of the plates, a comparison study has been done while the frequency results are in good agreements with the results of [49] and [50]. Fig. 2 shows the variation of the natural frequencies with the length of a square nanoplate for various non-local parameters. The frequency ratio is obtained by dividing the frequency of non-local plate by local one with

$\mu = 0$. Here one can see that with increasing the non-local parameter, the frequency ratio will decrease, this trend is reported for higher mode of vibration as seen in Table 2. Also, with enlarging the plate dimensions, in horizontal coordinate, the frequency ratio will increase, that means the effect of non-local parameter in small nanoplates are more significant.

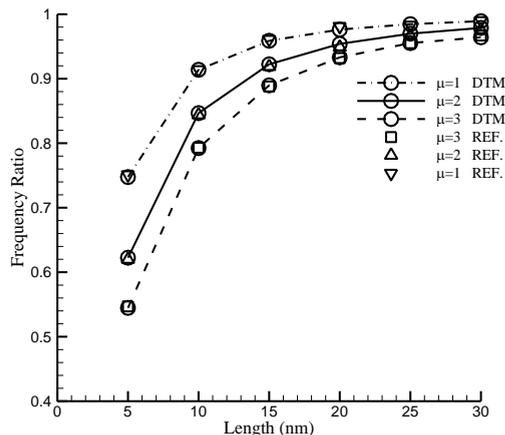


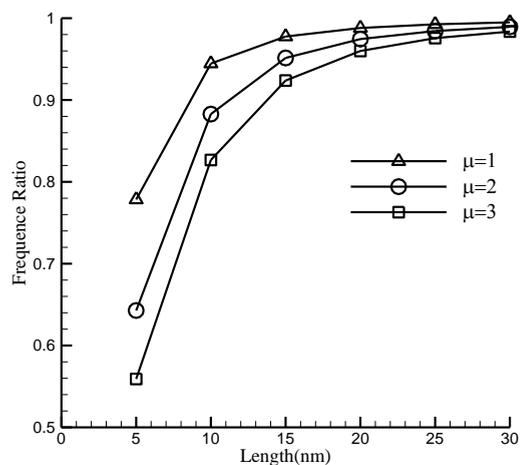
Fig. 2 Variation of SSSS natural frequencies ratio with the length of a square nanoplate for various non-local parameter

Table 2 Variation of natural frequencies of a square nanoplate for various non-local parameter

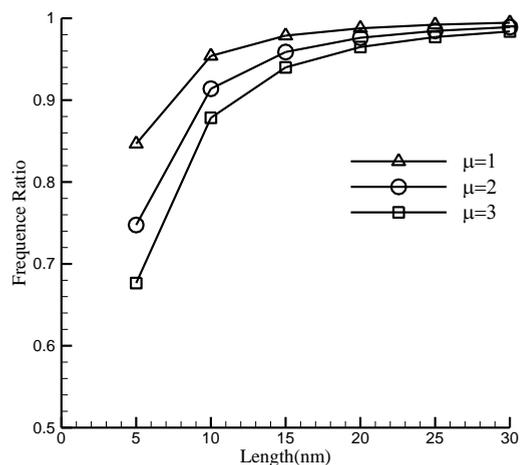
Frequencies	μ	third-order shear deformation theory [50]	DTM
ω_{11}	0	0.0936	0.0935
	1	0.0855	0.0854
	2	0.0792	0.0791
	3	0.0742	0.0741
	4	0.0699	0.0699
ω_{22}	5	0.0664	0.0663
	0	0.3455	0.3458
	1	0.2583	0.2585
	2	0.2151	0.2153
	3	0.1882	0.1884
ω_{33}	4	0.1694	0.1696
	5	0.1553	0.1555
	0	0.7008	0.7020
	1	0.4206	0.4213
	2	0.3284	0.3290
	3	0.2786	0.2790
	4	0.2461	0.2466
	5	0.2229	0.2233

4 RESULTS AND DISCUSSIONS

Frequency ratio for various length of the plate for different boundary conditions, namely SSCC, SSFF, SSSF, SSCF, and SSSC are depicted in Fig. 3. The value of non-local parameter is assumed to be 1, 2, and 3. As shown in this figure, the rate of frequency change with non-local parameter in different boundary conditions is more or less the same.

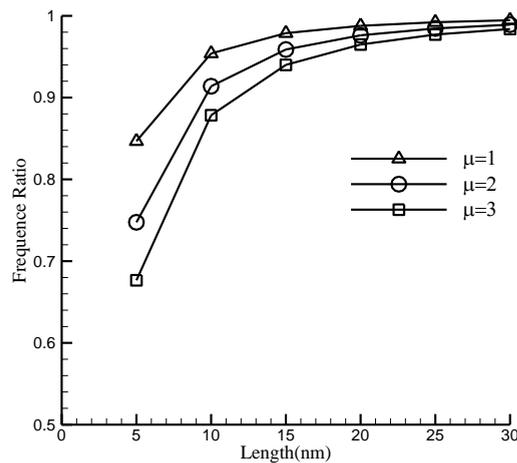


a)

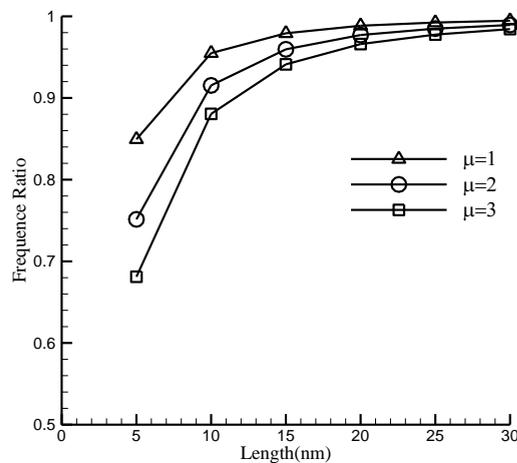


b)

SSSF, SSCC, SSSS, and finally SSSC, while the smallest frequency obtained for SSSC boundaries. Also, in higher non-local parameters, the change of frequency ratio is more meaningful in SSCC, SSSS, and SSSC against SSCF, SSFF, and SSSF. Fig. 5 and Fig. 6 show the influence of non-local parameter on the vibration of nanoplates with higher modes for SSSS and for other boundary conditions, respectively. Here, n and m are dedicated to the number of half-waves in x and for y direction of the nanoplate. In this figure, The value of non-local parameter is assumed to be 1. It can be seen that the frequency ratio decreases with increase in vibration modes. So, as can be seen in higher vibration modes, non-local parameter has a more significant role. Different boundary conditions here are examined.

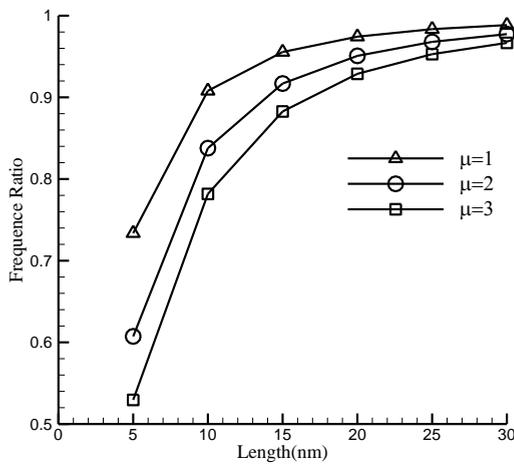


c)

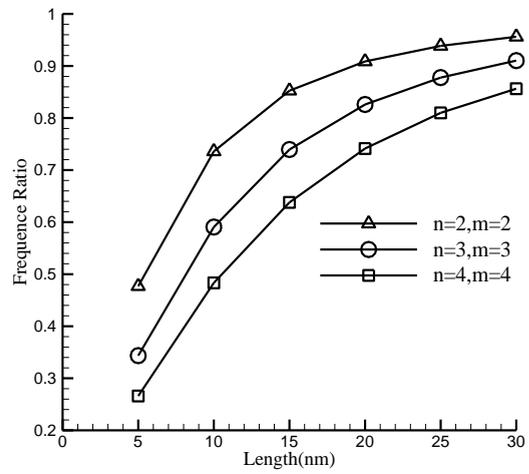


d)

Fig. 4 can give a better understanding about this frequency ratio. The same results for decreasing frequencies with enhancing non-local parameter observed here. Furthermore, with increasing non-local parameter μ , the frequency for different boundary conditions decrease in the order from SSCF to SSFF,



e)



a)

Fig. 3 Frequency ratio for various length of the plate for different boundary conditions, a) SSCC, b) SSCF, c) SSFF, d) SSSF, e) SSSC

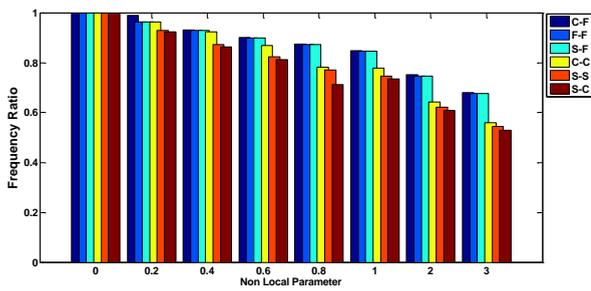
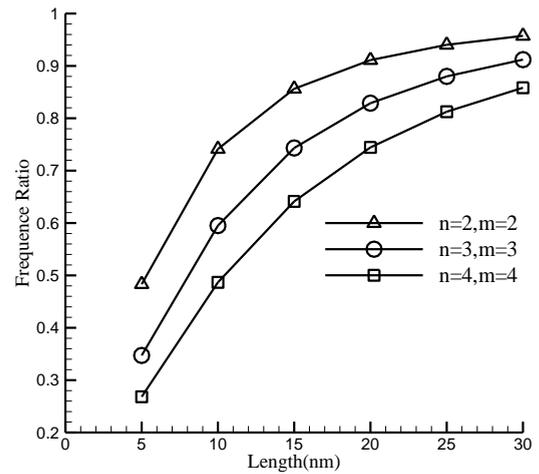


Fig. 4 Frequency ratio of the plate for various non-local parameter and boundary condition



b)

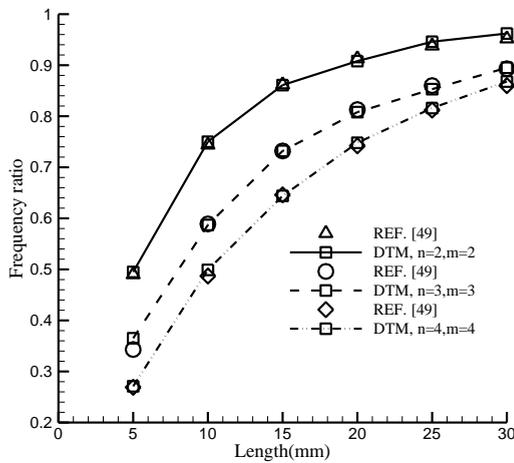
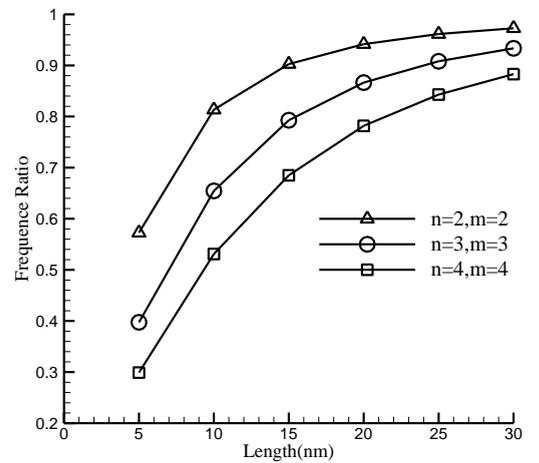


Fig. 5 Higher mode frequency ratio for various length of the plate for SSSS boundary condition



c)

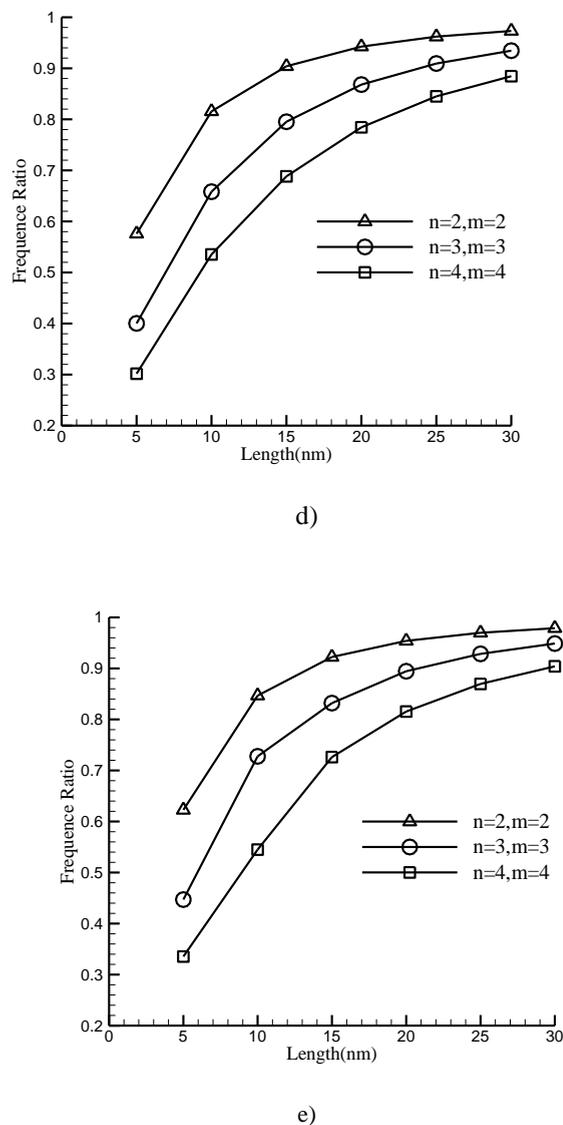


Fig. 6 Higher mode frequency ratio for various length of the plate for different boundary condition, a) SSSC, b) SSSC, c) SSFF, d) SSSF, e) SSCF

5 CONCLUSION

Main equation of motion of the thin classical plate is obtained using non-local theory of elasticity. This equation was solved with the aim of DTM method for different boundary conditions. Effects of non-local parameter, length, and boundary conditions are investigated for fundamental and higher vibration modes of nanoplates. In this paper, it is confirmed that frequency ratio of the nanoplate decreases with an increase in non-local parameter. Also, the significance of non-local parameter is more highlighted in smaller nanoplates and higher vibration modes.

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