A New Robust Strategy to Improve the Transient Dynamic of a Vehicle

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Abstract: In this paper for handling improvement and lateral stability increment of a four-wheeled vehicle a new robust active control system is proposed. First, to establish an accurate model of the vehicle, a fourteen-degrees-of-freedom nonlinear dynamic model is developed. Then, the nonlinear dynamic model is validated using CarSim software in a standard maneuver. Next, a new active steering control system was designed based on a simplified two-degrees-of-freedom dynamic mode to control the lateral motion of the vehicle. Two state variables, namely the vehicle’s yaw rate and the vehicle’s lateral velocity, are controlled using the control system. Also, the sliding mode control method is used to eliminate the error between the actual response and the desired response. Moreover, a complete stability analysis is presented based on the Lyapunov theory to guarantee closed-loop stability. Simulation results show that the controller is able to increase the vehicle’s maneuverability, especially during severe double lane change maneuver in which intense instability occurs. More investigations demonstrate that the proposed control system can considerably improve the vehicle’s path tracking under uncertainties.

Keywords: Handling, Path Tracking, Sliding Mode Control, Uncertainty


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1 INTRODUCTION

Handling improvement is a fundamental and significant issue for reducing the road accidents and has been a topic of considerable research for a long time. More investigations show that the lateral motions of the vehicle are controlled by steering control systems or braking control systems. According to an analysis by the National Highway Traffic Safety Administration, electronic stability control has helped to decrease the total number of accidents. Given the road condition and tire normal force, the lateral force of the tire is a function of the tire slip ratio and slip angle. Changing the steering angle will affect the vehicle lateral dynamics [1]-[6].

Many efforts have been done using active control systems to improve the lateral dynamic of the vehicle. Talebi et al. [7-9] optimized the rollover threshold of the vehicles for different velocities during steady state and transient maneuver. Nam et al. [10] proposed a robust yaw stability control based on active steering control. They used a two-degree-of-freedom control method by a disturbance observer to control the system for yaw stabilization. Moreover, the feedforward disturbance compensator was designed to compensate for the unexpected yaw moment caused by the difference between the torque of the left driving motor and the torque of the right driving motor. Kim et al. [11] designed an active steering control system for the rear wheels to increase the vehicle’s maneuverability. They used a three-degrees-of-freedom linear dynamic model to simulate the vehicle model. Moreover, they studied the effects of different parameters on the roll dynamic responses of the vehicle. Guvenc et al. [12] investigated a robust steering controller for yaw stabilization tasks in a driver-assist system. The yaw-stability-enhancing steering controller was designed in the parameter space to satisfy a frequency-domain mixed-sensitivity constraint. Zhang et al. [13] designed an active control system based on the quantitative feedback theory method. Nashar et al. [14] uses the LQR control method to develop an active steering control system. This control system neglects the important nonlinearities due to the suspension and tire characteristics that would be observed in a real vehicle at the high lateral accelerations discussed. In the paper by Chen et al. [15] when the longitudinal stability and the lateral stability of vehicle motion in the non-linear region were analyzed, the stability boundaries corresponding to different working conditions were obtained on the basis of the eigenvalues of the Jacobian matrix at each equilibrium point.

This research presents an active steering control system to improve the dynamic performance of the vehicle in critical conditions. This paper is organized as follows. First, a nonlinear dynamic model of a four-wheeled vehicle is developed. Then, the dynamic model is validated using CarSim software in a standard maneuver. In section 3, a new active steering control system is designed based on a simplified dynamic model to improve the vehicle’s dynamic performance. Also, the sliding mode control method is used for controller design to compensate uncertainties. In this controller, the steering angle of the front axle is used as the control input which makes the state variables to follow their desired values. Next, the performance of the control system is evaluated in standard maneuvers for different conditions. Finally, Conclusions are provided.

2 VEHICLE DYNAMIC MODELLING

In this section the vehicle model is described. The vehicle’s dynamic for the horizontal, lateral, vertical, roll, pitch and yaw motions can be presented by a fourteen-degrees-of-freedom nonlinear dynamic model. The schematic diagram of a nonlinear dynamic model is shown in “Fig. 1”. The steering angles $\delta_f$ of both front wheels are considered identical, a common assumption for high-speed cornering.

![Fig. 1 The fourteen-degrees-of-freedom nonlinear dynamic model [15].](image)

2.1. Equations of Motion

The governing equations for longitudinal, lateral and vertical, roll, pitch and yaw motions, as well as the rotational motion of each wheels are:

$$\dot{v}_x = \frac{\sum F_x}{m_\text{e}} - \theta v_z + v_y \dot{\psi}$$

(1)
\( v_y = \sum \frac{F_y}{m_t} + \phi v_x - v_x \dot{\psi} \)  \hspace{1cm} (2)
\( v_x = \sum \frac{F_x}{m_t} - \phi v_y + v_x \dot{\theta} \)  \hspace{1cm} (3)
\( \dot{\phi} = \frac{\sum M_y}{I_{xx}} + (I_{yy} - I_{xx}) \ddot{\psi} \) \hspace{1cm} (4)
\( \dot{\theta} = \frac{\sum M_y}{I_{yy}} + (I_{zz} - I_{xx}) \phi \dot{\psi} \) \hspace{1cm} (5)
\( \ddot{\psi} = \frac{\sum M_z}{I_{zz}} + (I_{xx} - I_{yy}) \phi \dot{\theta} \) \hspace{1cm} (6)
\( \dot{\omega}_1 = \frac{T_1 - R_w F_{t1}}{I_w} \) \hspace{1cm} (7)
\( \dot{\omega}_2 = \frac{T_2 - R_w F_{t2}}{I_w} \) \hspace{1cm} (8)
\( \dot{\omega}_3 = \frac{T_3 - R_w F_{t3}}{I_w} \) \hspace{1cm} (9)
\( \dot{\omega}_4 = \frac{T_4 - R_w F_{t4}}{I_w} \) \hspace{1cm} (10)

2.2. Tire Dynamics

One of the most importance stages to simulate vehicle dynamics is tyre modelling. In the linear model, due to the direct relation between the lateral force and the tyre side slip angle, the tyre normal forces are ignored. In this condition, the tyre potential is disregarded for slip prevention and its saturation. As far as the linear tyre model crosses the linear zone boundaries at high velocities, as well as high side slip angles, a nonlinear tyre model is considered. The traction and side forces acting on the articulated vehicle are generated at the contact path between tyre and road. The traction forces are generated for acceleration or braking and the side forces are necessary to adjust the direction of the vehicle. In this article, the Magic Formula model is used to simulate the tyre forces [17].

\[
\lambda = \frac{\mu F_{si}}{2} \left[ 1 - \varepsilon \frac{u_i \sqrt{S_i^2 + \tan^2 \alpha_i}}{(1 - S_i)} \right]
\]

\[
f(\lambda) = \begin{cases} 
\lambda(\lambda - 2) & \text{if } \lambda < 1 \\
1 & \text{if } \lambda > 1 
\end{cases}
\]

\[
F_{si} = \frac{C_u \tan \alpha_i}{1 - S_i} f(\lambda)
\]

\[
F_{ti} = \frac{C_u S_i}{1 - S_i} f(\lambda)
\]

2.3. Validation of Four-Wheeled Vehicle Model

In this paper to validate the vehicle dynamic model CarSim software is used [18]. According to “Fig. 2 and 3”, in order to verify the transient response of the developed model, a double lane change steering input was applied while the vehicle was running at a speed of 70 km/h. The steering angle is shown in “Fig. 4”.

The parameters of a passenger car are available [19]. Figures 5 to 7 show the comparison of the yaw rate, roll angle and lateral acceleration responses between the developed model and CarSim results during a transient
maneuver. The test was carried out on a dry road. As can be seen from “Figs. 5 and 6”, the responses of the vehicle model matched acceptably those of the CarSim test results. The comparative roll angle response is illustrated in “Fig. 7”. Moreover, although the roll angle curve is in fairly good agreement with the CarSim model results, some deviations are observed between the two during the period t = 5–7 s.

### 3 CONTROL SYSTEM DESIGN

To improve the vehicle’s handling and its lateral stability, the yaw rate and lateral velocity of the vehicle are controlled to follow their desired values. The main objective of the controller is to eliminate the error of the yaw rate and the lateral velocity of the vehicle between the actual values and their desired values. For this purpose, a new control system is developed based on a simplified dynamic model. In this paper, a conventional linear two-degrees-of-freedom dynamic model is developed. For such a model, the following set of assumptions is made to further idealize the vehicle motions:

- The simplified dynamic model has two degrees-of-freedom. The degrees of freedom are the yaw rate and the lateral velocity of the vehicle.
- In the dynamic model, the longitudinal velocity $v_x$ is assumed to be constant.
- The linear tire model is used in order to extract the lateral tire forces.

A vehicle’s handling dynamics in the horizontal plane are represented here by the single track, or bicycle model with states of lateral velocity at the center of gravity (CG) and yaw rate. Derivation of the equations of motion for the bicycle model follows from the force and moment balance:

$$m(v_y + v_x r) = F_{yr} + F_{yr} \cos(\delta)$$

$$I_{xz} \ddot{r} = F_{yr} l_1 \cos(\bar{\delta}) - F_{yr} l_2$$

Where $\delta$ is the steering angle, $v_x$ and $v_y$ are the longitudinal and lateral components of the CG velocity, $F_{yr}$ and $F_{yr}$ are the lateral tire forces front and rear, respectively, and $\alpha_f$ and $\alpha_r$ are the tire slip angles. In line with the small angle assumptions, one can have $\cos \delta = 1$ and the lateral tire forces of both axles can be expressed as the product of cornering stiffness $C_i$, and tire slip angle $\alpha_i$. In addition, the front and rear tire slip angles can be presented in the Appendix. In stationary turns, a definite relationship exists between the steering angle, the vehicle longitudinal velocity, and the yaw rate. This relationship is used to drive the desired yaw rate [20]:

$$r_d = \frac{v_x}{l \left(1 + \frac{K_{yz}}{l} v_x^2\right)} \delta$$

The equation of the system is written as follows.
\[
\begin{bmatrix}
m & 0 \\
0 & I_{zz}
\end{bmatrix}
\begin{bmatrix}
v_y \\
r
\end{bmatrix}
+ \begin{bmatrix}
m v_x r \\
l_1 F_y f - l_2 F_y r
\end{bmatrix} = \begin{bmatrix}
F_y f + F_y r \\
l_4 F_y f - l_2 F_y r
\end{bmatrix}
\]  \hspace{1cm} (15)

Where

\[A \dot{X} + C = u\]  \hspace{1cm} (16)

Where

\[
A = \begin{bmatrix}
m & 0 \\
0 & I_{zz}
\end{bmatrix}, \quad C = \begin{bmatrix}
m v_x r \\
0
\end{bmatrix}
\]

\[
u = \begin{bmatrix}
F_y f + F_y r \\
l_4 F_y f - l_2 F_y r
\end{bmatrix}, \quad X = \begin{bmatrix}
v_y \\
r
\end{bmatrix}
\]  \hspace{1cm} (17)

The tracking error is defined as:

\[e = X - X_d\]  \hspace{1cm} (18)

Therefore

\[e = \begin{bmatrix}
v_y \\
r
\end{bmatrix} - \begin{bmatrix}
v_y d \\
r_d
\end{bmatrix}\]  \hspace{1cm} (19)

The sliding surface is then selected as:

\[s = e\]  \hspace{1cm} (20)

Here, the sliding surface can be interpreted as the surface of the yaw rate error and the lateral velocity error between the vehicle and the reference model [20]. As \(s\) goes to zero, the control system can track the reference, the yaw rate and the lateral velocity perfectly. The sliding motion can be viewed as an average of the system dynamics on both sides of the sliding surface. By derivation from the sliding surface equation:

\[\dot{s} = 0 \rightarrow \dot{X} - \dot{X}_d = A^{-1}(C + u) - \dot{X}_d = 0\]

\[\ddot{u} = \dot{A}X_d + \dot{C}\]  \hspace{1cm} (21)

The control input is calculated as:

\[u = \ddot{u} - k \text{sgn}(s) = \dot{A}X_d + \dot{C} - k \text{sgn}(s)\]  \hspace{1cm} (22)

Where \(k\) is a positive parameter to be tuned in the controller design and \(\text{sgn}\) is the sign function, respectively. According to Eq. (17) we will have:

\[u = b F_y f\]  \hspace{1cm} (23)

Where

\[b = \begin{bmatrix}
1 & 1 \\
l_1 & -l_2
\end{bmatrix}\]  \hspace{1cm} (24)

In line with the small angle assumptions, one can have \(\cos \delta = 1\) and the lateral tire forces of both axles can be expressed as the product of cornering stiffness \(C_i\), and tire slip angle \(\alpha_i\):

\[F_y i = C_{ai} \alpha_i\]  \hspace{1cm} (25)

Establishing equalization of equations (23) and (25) the following equation is:

\[\alpha_i = \frac{b^{-1} u}{C_{ai}}\]  \hspace{1cm} (26)

The control law defined in equation (22) is discontinuous when crossing the sliding surface \(s = 0\) which may lead to undesirable chattering. To avoid this, the \(\text{sgn}\) function can be replaced by an approximation such as a saturation function according to:

\[\text{sat}(s/\varnothing) = \begin{cases}
s/\varnothing, & \text{if } |s| \leq \varnothing \\
\text{sgn}(s/\varnothing), & \text{if } |s| > \varnothing
\end{cases}\]  \hspace{1cm} (27)

Where \(\varnothing\) is the boundary layer thickness.

4 SIMULATION RESULTS

In this section, the robust performance of the designed control systems is investigated during steady-state and transient maneuvers.

4.1 Step Steer Maneuver

In this maneuver, the vehicle runs at an initial velocity of 70 km/h on a dry road with 0.7 friction coefficient and the steering input shown in “Fig. 8”.

![Fig. 8 The steering angle.](image)

The simulation results are shown in “Figs. 9 to 13” for controlled and uncontrolled states. As can be seen from “Fig. 9”, for uncontrolled condition, the vehicle is stable, however, the yaw rate of the vehicle deviates from its
desired value. According to “Fig. 10”, for uncontrolled condition, the vehicle’s lateral velocity increases significantly. Figure 9 shows that the active steering control system makes the yaw rate persuasion in an acceptable level. In the controlled case, the vehicle’s lateral velocity is limited in a narrow band.

However, the possibility of lateral instability reduces. Also, the trajectory of the vehicle is illustrated in “Fig. 11”, for controlled and uncontrolled conditions. According to Figure, the uncontrolled vehicle leaves its path, for the controlled condition, the tracking of the desired yaw rate leads to follow its desired trajectory. As can be seen from “Fig. 12”, the control system improves the lateral acceleration’s peak value and settling time considerably in comparison with uncontrolled mode. The control effort is shown in “Fig. 13”.

4.2. Double Lane Change Maneuver
In this maneuver, the vehicle runs on an icy road with a friction coefficient of 0.3 at the constant speed of 100 km/h and the steering input shown in “Fig. 14”. The dynamic responses are illustrated in “Figs. 15 to 19”. According to “Fig. 15”, the vehicle is severely unstable and leaves its path due to yaw instability occurrence. For controlled condition, the control system can perform the control task successfully. Moreover, in this state, the yaw rate tracks its desired value accurately.
According to “Fig. 16”, the lateral velocity of the vehicle reduces in comparison with uncontrolled state, significantly. In other words, by reducing the lateral velocity, the vehicle’s lateral stability increases. The vehicle trajectory is shown in “Fig. 17” at two conditions. For the uncontrolled state, the vehicle is unstable and deviates from its desired path while for the controlled vehicle no path departure is observed. According to “Fig. 18”, a noticeable reduction for the lateral acceleration’s peak value is observed for the controlled condition. The control input is shown in “Fig. 19". As can be seen from Figure, the curve of the steering angle has a smooth form.

Fig. 14  The steering angle.

Fig. 15  Yaw rate.

Fig. 16  Lateral velocity.

Fig. 17  Vehicle trajectory.

Fig. 18  Lateral acceleration.
5 CONCLUSION

Maneuverability enhancement is an important subject for the lateral dynamics of the vehicle. This paper proposes a new non-linear control system for simultaneous vehicle-handling and path-tracking improvement. First, a nonlinear dynamic model of the vehicle is proposed. Next, a robust control system is designed to improve the dynamic performance of the vehicle. The active steering control system is designed based on a simplified dynamic model and on the basis of sliding mode control method. Moreover, the control system performance is evaluated during steady state and transient maneuvers. The followings are the main measures and conclusions considered here:

- A fourteen-degrees-of-freedom nonlinear dynamic model is proposed. Then, the vehicle dynamic model is validated using CarSim software during the standard maneuver.
- The uncontrolled vehicle is stable in step steer maneuver, but the vehicle’s yaw rate deviates from its desired value.
- The active steering control system is used to improve the handling and lateral stability of the vehicle. In the control system, the yaw rate and the lateral velocity of the vehicle are considered as control variables which are targeted to track their desired values.
- The vehicle becomes severely unstable and deviates from its desired path during double lane change maneuver. In the controlled condition, the active steering system makes desired yaw rate persuasion at an acceptable level.
- The lateral velocity of the controlled vehicle is limited in a narrow band for both maneuvers.
- The simulation results confirmed that the controller not only improves the vehicle maneuverability, but also increases the lateral stability in the presence of uncertainties.

APPENDIX

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
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<tr>
<td>$C_{sf}$</td>
<td>4.05(6.88)</td>
<td>KN. m. s/rad</td>
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<tr>
<td>$C_{sr}$</td>
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<td>KN. m. s/rad</td>
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<td>m</td>
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<td>$h_2$</td>
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<td>$I_w$</td>
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<tr>
<td>$T_r$</td>
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</tbody>
</table>

NOMENCLATURE

- $C_{st}$: Front/ rear suspension damping constant
- $C_{st}$: Front/ rear tire damping constant
- $h_1$: Height of sprung mass
- $I_w$: Wheel moment of inertia
- $I_{xx}$: Roll moment of inertia
- $I_{yy}$: Pitch moment of inertia
- $I_{zz}$: Yaw moment of inertia
- $L_f$: Distance of the center of gravity from the front axle
- $L_r$: Distance of the center of gravity from the rear axle
- $M_s$: Vehicle sprung mass
- $M_{uf}$: Front unsprung mass
- $M_{ur}$: Rear unsprung mass
- $M_t$: Vehicle total mass
- $T_f$: Front track width
- $T_r$: Rear track width
- $\phi$: Roll angle
\[ \theta \] Pitch angle  
\[ \psi \] Yaw angle  
\[ \delta \] Steer angle  
\[ v_x \] Longitudinal velocity  
\[ v_y \] Lateral velocity  
\[ v_z \] Vertical velocity  
\[ F_x \] Tire's longitudinal force  
\[ F_y \] Tire's lateral force  
\[ r \] Yaw rate  
\[ c_a \] Cornering stiffness

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**REFERENCES**


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