

Quick and Effective Modal and Flutter Analyses for Low Aspect Ratio Wings

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Abstract: In the present work, an analytical study is proposed to investigate the flutter behavior of low-aspect-ratio wings in subsonic flow. An equivalent plate model is used for structural modelling of a semi-monocoque main wing, consisting of ribs, skins, and spars. Legendre polynomials are used in the Rayleigh-Ritz method as trial functions, and the first-order shear deformation theory is utilized to formulate the structural deformation. Boundary conditions are enforced by applying proper artificial springs. A doublet point method is used to calculate the unsteady aerodynamic loads. Chordwise pressure coefficient distribution at the tip and root of a rectangular wing oscillating in pitching motion is calculated. Flutter analysis is performed using the k method. Instead of using the computationally expensive finite element method, the proposed approach is intended to achieve purposes of quick modelling and effective analysis in free vibration and flutter analyses of low-aspect-ratio wings for preliminary design applications. The effects of aspect ratio on the flutter behavior of wings in subsonic flow are investigated. The obtained results are validated with the results available in the literature.

Keywords: Doublet Point Method, Equivalent Plate Model, Flutter, Low Aspect Ratio, Unsteady Aerodynamics

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1 INTRODUCTION

The field of aeroelasticity involves mutual interaction among the aerodynamic, inertia, and elastic forces. Flutter is a dynamic instability of an aeroelastic system, characterized by the interactions of aerodynamic loads and elastic deformations. It is an aeronautically important phenomenon since the onset of flutter may rapidly develop into structural failure [1]. Therefore, constructing an appropriate aeroelastic analysis algorithm for an intended purpose is very important.

The Finite Element Method (FEM) is a well-known method with wide applications in the structural analysis process and solving engineering problems. Since the structural parameters often change in the preliminary design of aircraft, modifying the Finite Element Models (FEMs) has many difficulties. The preparation time for FEM data may be unreasonable. A detailed Finite Element Analysis (FEA) for complex structures needs enormous amounts of CPU times and computation capacity [2]. In conceptual design or initial design stages of aircraft wing structures, equivalent plate analysis has been utilized as an alternative to the computationally expensive finite element analysis. In the equivalent plate modelling based on the Ritz method, the model characteristics are represented with trial functions, which require only a few numbers of the input data in comparison to a corresponding FEM [3].

Flutter problem has been noted in the preliminary design of modern aircraft. In the preliminary design, developing simplified equivalent models for simulating a complex structure is necessary to avoid troubles of complex experiments, detailed FEA, and complicated calculations. In the equivalent plate models, degrees of freedom are reduced, and only essential modes are used for flutter analysis. The main points of using the equivalent models are effective analysis, fast modelling, and accuracy. The Equivalent Plate Models (EPMs) were first proposed by Giles [2].

This method rapidly became a useful tool for structural analysis in designing. Giles [4-5] developed EPM in forms of trapezoidal plates using the Classical Plate Theory (CPT) and the Ritz method. Tizzi [6-7] proposed a method almost similar to Giles's method. In this method, the inner parts of wing structures (Ribs, spars, etc.) were neglected for simplification, but several trapezoidal segments in different planes (Such as tails, winglets, etc.) were considered. Livne [8] developed a more accurate EPM method than of Giles by using the First-Order Shear Deformation Theory (FSDT). Simple polynomial functions were used as the trial function to define the structural deformations. Live et al. [9-10] further developed the EPM method to analyze geometrically nonlinear wing box structures. They selected simple-polynomials as trial functions; therefore, it was easy to encounter numerical ill-

conditioning problems, and the order of the polynomials was limited. Kapania et al. [11] used orthogonal Chebyshev polynomials in the Rayleigh-Ritz method as the trial functions to obtain EPM, so that ill-conditioning problems could be avoided. They used numerical integration so that CPU time was much longer. Later, Kapania and Liu [12] proposed an almost similar method using Legendre polynomials as the trial functions.

The past EPM method has been widely used in the preliminary design stage of aircraft structures in recent years. In these researches, the different kinds of plate units (Such as skin, rib web, spar web) of the wing composed of spars, skin, and ribs, are distinguished in modelling. That is, the integrals which must be solved are computed in different parts, respectively. It needs intricate pre-processing work. Krishnamurthy et al. [13], [14] presented EPM that could be used to design a wind tunnel model to match the stiffness characteristics of a wing box while satisfying strength-based requirements. EPM technique is also presented to investigate the static and dynamic response of a wing with damage.

Na and Shin [15] developed an equivalent plate method similar to that of Livne for structural modelling of low-aspect-ratio composite wings with a control surface. Tang et al. [16] presented a method of equivalent simplification, using equivalent-plate models. It was based on the assumption that the different plate units (such as rib web, beam web, skin) are not distinguished in modelling, which led to avoid the complex pre-processing and made it more generalized. Their method was based on the first-order shear deformation theory of plates. As trial functions, Legendre polynomials were applied in The Ritz method. The EPM was used for calculation of mass and stiffness matrices. Calculations of aerodynamic load and flutter analysis were performed using ZAERO software. The results of free vibration and flutter analyses obtained using EPM agreed well with those obtained by the FEM.

Aerodynamic calculations have a vital contribution to flutter analysis because structural deformations affect aerodynamic loads. The effects of elastic deformations at low flight speed are small. At high flight speed, elastic deformation can cause instability in a wing, or rendering a control surface ineffective, or even control reversal [17] (pp. 186-187).

Several researchers have presented unsteady aerodynamic theories for deriving the pressure distribution on a thin finite wing in subsonic flow. Kussner [18] proposed an integral equation. The mode function methods and the direct element methods are two principal categories that include all available methods. Watkins et al. [19] improved the mode function method practically. Redman and Rowe [20] obtained the unsteady pressure distribution on wings with control surfaces successfully. The doublet lattice method is a common discrete-element method type [21],

[22-23]. This well-known method can be used readily to complex wing configurations. The existence of an inconsistency in the steady-state part is the disadvantage of this method [21]. Ueda and Dowell [24] formulated a doublet point method for calculating unsteady aerodynamic loads in subsonic flow on harmonically oscillating thin wings. This method is based on the concept of concentrated loads. In other words, on aerodynamic elements, the lift distribution is replaced by concentrated lift forces.

Aeroelastic phenomena were solved with different theories. Hollowell and Dugundji [25] have investigated an analytical solution for aeroelastic analysis of unswept rectangular wings. The wings were simulated as plates with torsion-bending stiffness coupling. The obtained results by the analytical solution were verified by the experiment. The Rayleigh-Ritz method was used for calculation of mass and stiffness matrices, and unsteady two-dimensional aerodynamic theory was used for calculation of aerodynamic loads in incompressible flow. The U-g method was used for divergence and flutter analyses, and the obtained results were compared to the results of low-speed wind tunnel tests. They concluded that negative stiffness coupling makes divergence in a wing, while positive coupling delays the happening of stall flutter. Vepa [26] proposed an EPM based on the first-order shear deformation theory for an anisotropic plate. Also, he performed the aeroelastic analysis of a wing using EPM.

Shokrollahi et al. [27] used the first-order shear deformation plate theory combined with a supersonic Mach box method to study flutter speeds of a low aspect ratio trapezoidal structurally hybrid wing from low supersonic velocities to moderate velocities. The main objective of this work was the study of the effect of composite to metal spanwise length ratio on flutter speeds. The effect of fiber orientation with various geometric parameters such as sweep angle and taper ratio was investigated. They concluded that for specified geometric parameters, there exists an optimum metal to composite proportion at which the flutter speed is maximum. Also, it was realized that for a specified taper ratio or sweep angle, there was a maximum flutter speed corresponding to an optimum fiber orientation in the composite part. Babin et al. [28] presented a 2D plate model based on the fluid-structure interaction method and classical plate theory for the aeroelastic analysis of low aspect ratio composite wings. The supersonic loads and domain model were defined by first-order piston and Hamilton principle theories. By using the differential quadrature method literature, natural frequencies and flutter speeds were analyzed and verified. They concluded that the flutter characteristics are strongly dependent on cross-ply laminates. The effects of the aspect ratio on supersonic flutter performance of laminated wings were studied.

By studying the previously published papers, it can be observed that there are many methods presented for aerodynamic, structural, and aeroelastic analyses. Also, it can be concluded that appropriate selection and combination of these methods to each other to achieve an aeroelastic algorithm for an intended purpose is essential. In other words, a method may be suitable for a use, whereas it is not suitable for another use.

In the present work, an equivalent plate method is utilized for structural modelling of a semi-monocoque main wing, consisting of spars, skins, and ribs. Legendre polynomials are used in the Rayleigh-Ritz method as trial functions, and the first-order shear deformation theory is used to formulate the structural deformation. Boundary conditions are enforced by applying proper artificial springs. A doublet point method is utilized to calculate the unsteady aerodynamic loads. Chordwise pressure coefficient distribution at the tip and root of a rectangular wing oscillating in pitching motion is calculated. Flutter analysis is performed using the k method.

The effect of aspect ratio on the flutter behavior of low-aspect-ratio wings in subsonic flow is investigated. The obtained results are validated with the results available in the literature. This structural formulation has not been so far combined with the doublet point method, and it has not been utilized for an aeroelastic analysis by previously mentioned references. These methods have been used to achieve a new algorithm for flutter and modal analyses of low-aspect-ratio wings in the preliminary design stage instead of using the computationally expensive FEM. This new algorithm is very rapid and effective and requires only a few input data in comparison to a corresponding finite element model. Also, its runtime is smaller than the runtime of FEM. Therefore, effective analysis and fast modelling is the primary purpose of this article.

2 STRUCTURAL MODELLING OF WING USING EQUIVALENT PLATE METHOD

The transverse deflection can be written as follows:

$$w_0(\xi, \eta, t) = \{b_3(\xi, \eta)\}^T \{q_3(t)\} \quad (1)$$

Where, $\{b_3(\xi, \eta)\}$ is the vector of the trial functions in Ritz method and given in [12]:

$$\{b_3(\xi, \eta)\}^T = \{B_1(\xi)B_1(\eta), B_1(\xi)B_2(\eta), \dots, B_M(\xi)B_N(\eta)\} \quad (2)$$

In which B_i can be chosen to be the Legendre polynomials. ξ and η are transformed coordinates.

The skew trapezoidal plate is transformed into a square region with vertices having values ranging from -1 to 1. Legendre polynomials do not satisfy the plate boundary conditions and boundary conditions are enforced by applying proper artificial springs [12]. $\{q_3(t)\}$ is the Rayleigh-Ritz coefficients vector that is time-dependent. The equation of motion is obtained from Hamilton's principle and given in [12]:

$$[M]\{\ddot{q}\} + [K]\{q\} = \{F_{z0}\} \quad (3)$$

Where, $[M]$ and $[K]$ are the mass and stiffness matrices, respectively. $\{F_{z0}\}$ is the column vector of the aerodynamic load on the wing, and $\{q\}$ is the Rayleigh-Ritz coefficients vector. By adding all of the stiffness and mass matrices corresponding to each component, the stiffness and mass matrices of the entire wing can be calculated [12]:

$$[K] = [K_{skin}] + [K_{sapr}] + [K_{rib}] \quad (4a)$$

$$[M] = [M_{skin}] + [M_{sapr}] + [M_{rib}] \quad (4b)$$

All the equations used for structural modelling and calculating the stiffness and mass matrices of a semi-monocoque main wing, consisting of spars, skins, and ribs, are presented in [12].

3 UNSTEADY AERODYNAMIC FORMULATION

The subsonic unsteady aerodynamic loads on two-dimensional wings are calculated by the doublet point method. In the doublet point method, the distributed aerodynamic loads are replaced by concentrated loads vector $\{F_z\}$. The doublet point method can readily be coupled with aeroelastic analysis methods to perform flutter and divergence analysis. The relation between amplitudes of pressure distributions and their upwash velocity on oscillating lifting surfaces is as follow [18], [24], [29] (pp. 246):

$$v_i(x, y) = \frac{1}{8\pi} \iint_S \Delta p(\xi, \eta) K(x_0, y_0) d\xi d\eta \quad (5)$$

As an assumption, the lifting surface is in the x - y plane ($z = 0$). S is the region of the wing area, and the non-dimensional pressure coefficient Δp is written as follows:

$$\Delta p = \frac{-p'_+ + p'_-}{0.5\rho_\infty u_\infty^2} \quad (6)$$

Where, p'_- and p'_+ are disturbance pressure of the lower and upper surfaces of a wing, respectively. Also, ρ_∞ and u_∞ are density and velocity of the uniform flow, respectively. The Kernel function $K(x_0, y_0)$ in "Eq.(5)" is defined as:

$$K(x_0, y_0) = e^{-ikx_0} \left[\frac{Me^{ikx}}{R\sqrt{x^2 + r^2}} + B(k, r, x) \right] \quad (7)$$

The parameters of "Eq. (7)" are presented in [24]. The wing planform is partitioned into element surfaces so that the two side edges of each element are parallel to the uniform flow. The element surface has a width of $2\sigma_i$ and the area of Δ_i . In the doublet point method, the lift distribution on the i^{th} element surface is replaced by the equivalent concentrated load at the doublet point (ξ_i, η_i) . The upwash distribution on the i^{th} element surface is replaced by the equivalent upwash at the upwash point (x_i, y_i) . By considering these assumptions, the upwash of the i^{th} element surface in "Eq. (5)" is discretized and can be written as [24]:

$$v_i(x_i, y_i) = \frac{1}{8\pi} \sum_{j=1}^N \Delta p(\xi_j, \eta_j) \Delta_j K(x_i - \xi_j, y_i - \eta_j) \quad (8)$$

, ($i = 1, 2, \dots, N$)

Equation (8) can be rewritten as:

$$\{v_i\} = [D]\{c_p\} \quad (9)$$

Where, $[D]$ is the aerodynamic influence coefficients matrix, $\{v_i\}$ is the velocities induced at (x_i, y_i) , and $\{c_p\}$ is the vector of pressure coefficient which is representative of the pressure coefficient at (ξ_i, η_i) and defined as follows:

$$v_i = \{v_{ii}\} = \{v_i(x_i, y_i)\} \quad (10)$$

$$D = [d_{ij}] = \frac{\Delta_j}{8\pi} K(x_i - \xi_j, y_i - \eta_j) \quad (11)$$

$$c_p = \{c_{pj}\} = \{\Delta p(\xi_j, \eta_j)\} \quad (12)$$

The upwash vector for the i^{th} element is calculated as [24]:

$$v_{ii} = \frac{\partial}{\partial x} w_0(x_i, y_i) + ikw_0(x_i, y_i) \quad (13)$$

Where, $w_0(x_i, y_i)$ is the transverse deflection of the mid-plane at the upwash point of the i^{th} element surface. The induced velocity v_{I_i} derived from equation “Eq. (1)” and “Eq. (13)” is as follows:

$$v_{I_i} = ([v_{IR}]_i + i[v_{II}]_i) \{q_3\} \quad (14a)$$

$$[v_{IR}]_i = \left\{ \frac{\partial b_3}{\partial x}(x_i, y_i) \right\} \quad (14b)$$

$$[v_{II}]_i = k \{b_3(x_i, y_i)\} \quad (14c)$$

The induced velocities for all elements are calculated as follows:

$$\{v_I\} = [v_{IR}] \{q_3\} \quad (15a)$$

$$[v_{IT}] = [v_{IR}] + i[v_{II}] \quad (15b)$$

By substituting $[v_{IT}]$ from “Eq. (15b)” into “Eq. (9)”, $\{c_p\}$ is obtained as:

$$\{c_p\} = [D]^{-1} [v_{IT}] \{q_3\} \quad (16)$$

The transverse aerodynamic load at the i^{th} element surface can be calculated by [30]:

$$F_{ai} = \frac{1}{2} \rho U^2 S_i c_{pi} \quad (17)$$

Where, U is the uniform airflow speed, ρ is the density of air, and S_i is the area of element surface i . The transverse load vector for all element surfaces is obtained as follows:

$$\{F_a\} = \frac{1}{2} \rho U^2 [S] \{c_p\} \quad (18)$$

The main diagonal of the matrix $[S]$ consists of elements areas. Substituting “Eq. (16)” into “Eq. (18)” gives:

$$\{F_a\} = \frac{1}{2} \rho U^2 [Br] \{q_3\} \quad (19)$$

Where, $[Br]$ is defined as:

$$[Br] = [S][D]^{-1} [v_{IT}] \quad (20)$$

Due to the symmetry, only the transverse aerodynamic loads on the half wing are calculated. Using the energy method, the concentrated transverse aerodynamic load vector at the doublet point of element surface i is derived as [12]:

$$\{F_z\}_i = F_{ai} \{b_3(\xi_i, \eta_i)\}^T \quad (21)$$

For all elements of the half wing, “Eq. (21)” can be changed into as:

$$\{F_z\} = \sum_{i=1}^{N_e/2} \{F_z\}_i = [b_3][F_a] \quad (22)$$

Where, $[b_3]$ is a matrix that its columns are the column vectors of Ritz trial functions at the doublet points. Also, the parameter N_e is the total number of element surfaces. Substituting “Eq. (19)” into “Eq. (22)” yields:

$$\{F_z\} = \frac{1}{2} \rho U^2 [b_3][Br] \{q_3\} \quad (23)$$

Finally, the equation of motion of the wing is obtained as:

$$[M_w] \{\ddot{q}_3\} + [K_w] \{q_3\} = \frac{1}{2} \rho U^2 [b_3][Br] \{q_3\} \quad (24)$$

Where, $[K_w]$ and $[M_w]$ are sub-matrices of the stiffness and mass matrices of the wing corresponding to the transverse deflection [12], [30].

4 FLUTTER ANALYSIS

Wing motion is assumed harmonic, so $\{q_3\}$ can be written as follows:

$$\{q_3\} = \{\bar{q}_3\} e^{i\omega t} \quad (25)$$

Where, $\{\bar{q}_3\}$ is the amplitude of the Ritz coefficients vector, and ω is the oscillation frequency of the wing. The reduced frequency k is defined as follows [31] (pp. 549):

$$k = \frac{b\omega}{U} \quad (26)$$

By substituting “Eq. (25) and Eq. (26)” into “Eq. (24)” yields:

$$[K_w] \{\bar{q}_3\} - \omega^2 ([M_w] + [\bar{L}]) \{\bar{q}_3\} = 0 \quad (27)$$

Where, $[\bar{L}]$ is the aerodynamic matrix:

$$[\bar{L}] = \rho \frac{b^2}{2k^2} [b_3] [Br] \quad (28)$$

The k method is utilized to perform flutter analysis [31] (pp. 545-551). In this method, artificial structural damping g is added to the equation of motion, and flutter occurs when $g=0$ [25].

The eigenvalue problem obtained from “Eq. (27)” is written as:

$$((1+ig)[K_w] - \omega^2([M_w] + [\bar{L}]))\{\bar{q}_3\} = 0 \quad (29a)$$

$$(Z[K_w] - [\bar{B}])\{\bar{q}_3\} = 0 \quad (29b)$$

Where:

$$[\bar{B}] = [M_w] + [\bar{L}] \quad (30)$$

And:

$$Z = \frac{(1+ig)}{\omega^2} \quad (31)$$

The complex matrix $[\bar{L}]$ is calculated for each value of reduced frequency k , and the complex eigenvalues Z is obtained from “Eq. (29b)”.

The ω , g , and U parameters are calculated as follows:

$$\omega = \sqrt{\frac{1}{\text{Re}(Z)}}, g = \frac{\text{Im}(Z)}{\text{Re}(Z)}, U = \frac{\omega b}{k} \quad (32)$$

The value of the parameters ω and U at $g=0$ are the flutter frequency and flutter velocity, respectively [25].

5 NUMERICAL RESULTS AND DISCUSSION

The written MATLAB codes for vibration, aerodynamic, and aeroelastic analyses are validated. The chordwise pressure coefficient distribution on a rectangular wing in unsteady flow is calculated. A free vibration analysis of a semi-monocoque main wing, consisting of ribs, skins, and spars, is performed. Also, flutter analyses of rectangular and trapezoidal plate wings are implemented. The effect of aspect ratio on the flutter behavior of wings in subsonic flow is investigated. The obtained results are verified with the results available in the literature.

5.1. Validation of The Written Aerodynamic Code

In order to validate the written aerodynamic code, a rectangular wing 2 meters by 4 meters oscillating in pitching motion around its mid-chord is used.

Real and Imaginary parts of the chordwise pressure coefficient distribution (C_{PaR} and C_{PaI}) at the root and tip of the rectangular wing are shown in “Figs. 1-2”, respectively ($M=0$, $k=1$, $AR=2$, $N_x=5$, $N_y=10$).

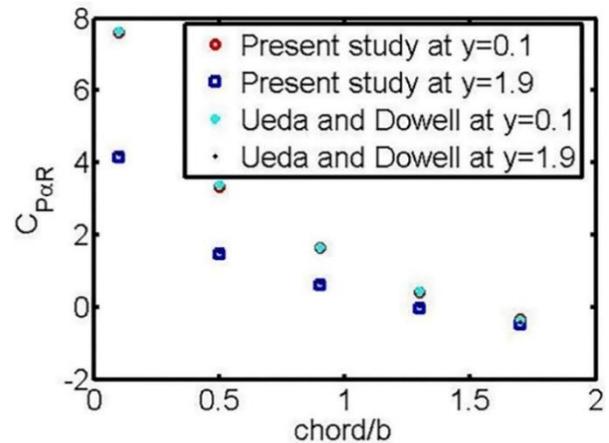


Fig. 1 Real part of the chordwise pressure coefficient distribution at the root and tip of the rectangular wing.

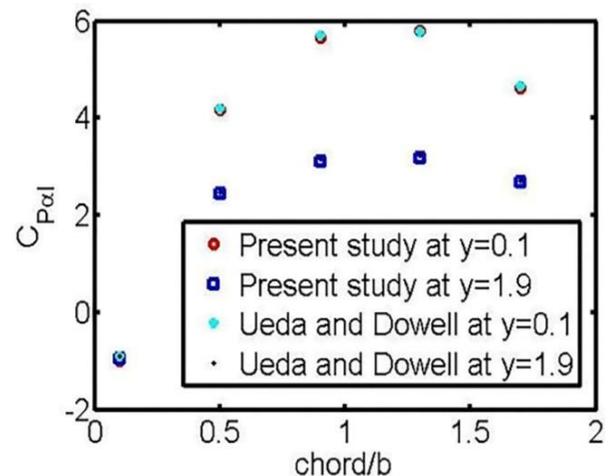


Fig. 2 Imaginary part of the chordwise pressure coefficient distribution at the root and tip of the rectangular wing.

Where, N_x and N_y are the chordwise number of elements and one-half of the spanwise number of elements, respectively. Also, M , AR , and α are Mach number of the uniform flow, aspect ratio of the wing, and angle of attack, respectively. The parameters b , C_{PaR} , and C_{PaI} are semi-chord, real, and imaginary parts of the chordwise pressure coefficient distribution, respectively.

The obtained results are similar to the results presented in [24].

5.2. Free Vibration Analysis of a Low Aspect Ratio Semi-monocoque Main Wing

The geometric and material parameters for the half-span representation of an aircraft wing are given as follows: span=4.88 m, root width=1.83 m, tip width=0.914 m, sweep angle (leading edge) =30 deg, thickness =0.0457 m, mass density $\rho = 2700 \text{ kg/m}^3$, Young’s modulus $E=70.7 \text{ GPa}$, and Poisson’s ratio $\nu = 0.3$. The wing is clamped at the root. The wing has ten ribs and four spars distributed uniformly. Other particulars of the wing are presented in “Table 1”.

Table 1 Component properties of the wing

Structure	Properties, mm
skin	Thickness: 3
rib/spar cap	Height: 5 Width: 9.47
rib/spar web	Thickness: 1.47

Natural frequencies (Hz) of the semi-monocoque wing with NACA0021 aerofoil are displayed in “Table 2”.

Table 2 Natural frequencies of the semi-monocoque wing with NACA0021 airfoil, Hz

Mode Number	Nastran [32]	Equivalent plate method [32]	Present study
1	12.91 Bending	13.06 Bending	13.48 Bending
2	47.73 Inplane	-	46.20 Inplane
3	56.57 Bending	60.77 Bending	62.64 Bending
4	68.12	-	107.79

5.3. Flutter Performance of Rectangular Plate Wings

The structural and air properties taken for this examination of a rectangular plate wing fluttering in the air are listed below:

$E = 0.4 \times 10^{10} \text{ N/m}^2$, $\nu = 0.25$, $G = 0.4 E$
 $\rho_s = 1500 \text{ kg/m}^3$, $t_s = 0.016 \text{ mm}$, $\rho_a = 0.45908 \text{ kg/m}^3$,
 Sound speed = 303.2 m/s

Where, ν , G and E are the Poisson's ratio, the shear modulus, and Young's modulus, respectively. Also, ρ_s , t_s , and ρ_a are the density, thickness of the wing, and the air density, respectively. In “Fig. 3”, the effect of aspect ratio AR on the flutter boundaries are illustrated for rectangular isotropic plate wings.

Where, V_F is the flutter speed, and V_R is the divergence speed at $AR = 4$. Figure 3 shows the isotropic case where the flutter speeds are normalized by the divergence speed at $AR = 4$. It can be seen that the flutter speeds increase monotonically as AR decreases. The increasing rates of flutter speeds are substantial when AR is less

than 1.5. The results shown in “Fig. 3” have good agreement with the results presented in [1].

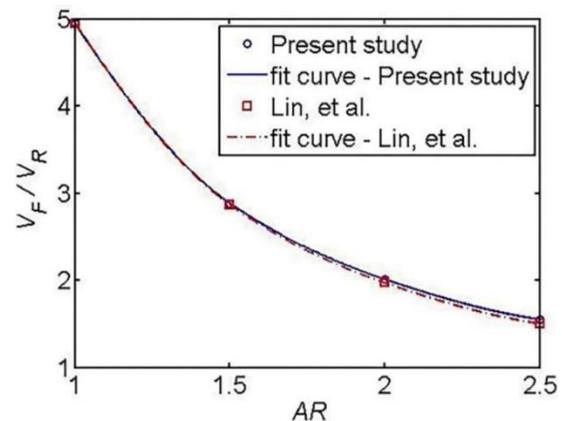


Fig 3 Normalized flutter speeds V_F/V_R vs. aspect ratio AR for isotropic wing ($V_R = 38.02 \text{ m/s}$).

5.4. Vibration and Flutter Analyses of a Trapezoidal Plate Wing

The material and geometric parameters for the half-span representation of a trapezoidal plate wing are given as follows:

span =0.5 m, root width =0.3 m, tip width =0.1 m, sweep angle =65.5 deg, thickness =0.5 mm, mass density $\rho = 7800 \text{ kg/m}^3$, Young’s modulus $E = 210 \text{ GPa}$ and Poisson’s ratio $\nu = 0.3$. The wing is clamped at the root. Air density is $\rho_a = 1 \text{ kg/m}^3$.

Natural frequencies of the trapezoidal plate wing are presented in “Table 3”.

Table 3 Natural frequencies of the trapezoidal plate wing, Hz

Mode Number	NASTRAN [33]	Present study	Error
1	2.1	2.11	0.48 %
2	10.24	10.27	0.3 %
3	14	14.1	0.7 %
4	26.36	26.7	1.3 %

The obtained flutter speed (U_F) is presented in “Table 4” and has good agreement with the results presented in [33].

Table 4 Flutter speed of the trapezoidal plate wing, m/s

Parameter	U_F , m/s
Present study	21.83
NASTRAN [33]	22
Error	0.8 %

6 CONCLUSION

An analytical approach is proposed to investigate the flutter behavior of low-aspect-ratio wings in subsonic flow. A doublet point method is utilized to calculate the unsteady aerodynamic loads on the wing. An equivalent plate method is used for structural modelling of a semi-monocoque main wing, consisting of ribs, skins, and spars. Boundary conditions are enforced by applying proper artificial springs. Legendre polynomials are used in the Rayleigh-Ritz method as trial functions, and the first-order shear deformation theory is used to formulate the structural deformation. Flutter problem is solved using the k method. Attaining quick modelling and effective analysis in free vibration and flutter analysis of low-aspect-ratio wings for the preliminary design applications are objectives of the present approach. Therefore, this approach can be applied for design or optimization calculations.

Chordwise pressure coefficient distribution at the tip and root of a rectangular wing oscillating in pitching motion is calculated. Vibration and flutter analyses of a trapezoidal plate wing are performed. The effect of aspect ratio on the flutter behavior of a rectangular wing in subsonic flow is investigated. The obtained results draw the following conclusions: flutter speeds increase monotonically as aspect ratio decreases. The present approach provides results that approximately match those from FEM, Nastran software, and other equivalent plate methods.

This approach is a quick and effective aeroelastic tool for the preliminary design stage of low-aspect-ratio wings because it requires only a few numbers of the input data and its computer code has a short runtime.

The proposed approach will be developed for flutter analysis of a low-aspect-ratio wing with a control surface and a winglet.

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